Particle Physics
The Standard Model
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Part IX

Cabibbo Kobayashi Maskawa

- The Particles
- $W^{\pm}$ couples to $SU(2)_L$ doublets
- Z°: no FCNC (Z° cannot change flavor just like γ)
- Assumption MassEigenstates=EWEigenstates
- Why?

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$$

$$\begin{matrix} u_R & c_R & t_R \\ d_R & s_R & b_R \\ e_R & \mu_R & \tau_R \end{matrix}$$

$$\begin{matrix} \gamma \\ g \\ W^{\pm}, Z^{\circ} \\ H \end{matrix}$$

- Another perspective: meson lifetimes
- Weak decays of mesons differ by orders of magnitude?
- How can the s decay weakly? $m_{\mathrm{c}} > m_{\mathrm{s}}$?
- Keep leptons untouched
- Introduce the CKM matrix

Properties of the c

$$
\begin{aligned}
\mathrm{m}_0 &= 1.27\mathrm{GeV} \\
\tau &= (1.040 \cdot 10^{-12})\mathrm{s} \quad c\bar{d} \\
c\tau &= 311.8\mu\mathrm{m}
\end{aligned}
$$

Properties of the s

$$
\begin{aligned}
\mathrm{m}_0 &= 100 \pm 25\mathrm{MeV} \\
\tau &= (1.24 \cdot 10^{-8})\mathrm{s} \quad u\bar{s} \\
c\tau &= 3.7\mathrm{m}
\end{aligned}
$$

Definition

d is the mass Eigenstate
d' is the isospin partner of u
V: unitary 3×3 matrix $\mathrm{V}^\dagger \mathrm{V} = 1_3$
Cannot simplify: masses not equal

$$
\begin{aligned}
\mathcal{L}_{Yuk} &= -\bar{\mathrm{u}} m_{\mathrm{u}} \mathrm{u} - \bar{\mathrm{c}} m_{\mathrm{c}} \mathrm{c} - \bar{\mathrm{t}} m_{\mathrm{t}} \mathrm{t} - \bar{\mathrm{d}} m_{\mathrm{d}} \mathrm{d} - \bar{\mathrm{s}} m_{\mathrm{s}} \mathrm{s} - \bar{\mathrm{b}} m_{\mathrm{b}} \mathrm{b} \\
&= -\begin{pmatrix} \bar{\mathrm{u}} & \bar{\mathrm{c}} & \bar{\mathrm{t}} \end{pmatrix} \begin{pmatrix} m_{\mathrm{u}} & 0 & 0 \\ 0 & m_{\mathrm{c}} & 0 \\ 0 & 0 & m_{\mathrm{t}} \end{pmatrix} \begin{pmatrix} \mathrm{u} \\ \mathrm{c} \\ \mathrm{t} \end{pmatrix} \\
&\quad -\begin{pmatrix} \bar{\mathrm{d}} & \bar{\mathrm{s}} & \bar{\mathrm{b}} \end{pmatrix} \begin{pmatrix} m_{\mathrm{d}} & 0 & 0 \\ 0 & m_{\mathrm{s}} & 0 \\ 0 & 0 & m_{\mathrm{b}} \end{pmatrix} \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \\ \mathrm{b} \end{pmatrix} - \begin{pmatrix} \bar{\mathrm{d}}' & \bar{\mathrm{s}}' & \bar{\mathrm{b}}' \end{pmatrix} \mathrm{V} \begin{pmatrix} m_{\mathrm{d}} & 0 \\ 0 & m \\ 0 & 0 \end{pmatrix}
\end{aligned}
$$

Properties of V

- Cabibbo, Kobayashi, Maskawa
- V complex: $3 \times 3 \times 2$
- $\mathrm{V}\mathrm{V}^\dagger = 1_3$: 9 constraints
- 5 phases absorbed
- 3 real mixing angles, 1 complex phase

Cabibbo

(2 generations):

$$\begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

Wolfenstein

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Diagonal entries dominate

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Charged Current

$$\begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \gamma^\mu \begin{pmatrix} {d_L}' \\ {s_L}' \\ {b_L}' \end{pmatrix}$$
$$= \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \gamma^\mu \mathrm{V} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

Decays

$$\begin{array}{llll} \mathrm{d} & \rightarrow & \mathrm{u} + \mathrm{W}^- & V_{11} \\ \mathrm{s} & \rightarrow & \mathrm{u} + \mathrm{W}^- & V_{12} \\ \mathrm{b} & \rightarrow & \mathrm{u} + \mathrm{W}^- & V_{13} \end{array}$$

- $V_{11} = 0.98$, $V_{12} = 0.2$
- charmed mesons:
  $V_{11}^2 G^2 \rightarrow 0.96 G^2$
- strange mesons:
  $V_{12}^2 G^2 \rightarrow 0.04 G^2$ longer lifetime

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Neutral Current

Neutral currents couple to right- and left-handed quarks:

$$\begin{pmatrix} \overline{d}' & \overline{s}' & \overline{b}' \end{pmatrix} \gamma^\mu \left(-\frac{1}{2}\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_W\right) \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

$$= \begin{pmatrix} \overline{d} & \overline{s} & \overline{b} \end{pmatrix} V^\dagger \gamma^\mu \left(-\frac{1}{2}\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_W\right) V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$= \begin{pmatrix} \overline{d} & \overline{s} & \overline{b} \end{pmatrix} V^\dagger V \gamma^\mu \left(-\frac{1}{2}\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_W\right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$V^\dagger V = 1_3$: does not have a Lorentz index, only family index

And the up-type sector?

$$\begin{aligned}
&\begin{pmatrix} \bar{u}_L{}' & \bar{c}_L{}' & \bar{t}_L{}' \end{pmatrix} \gamma^\mu \begin{pmatrix} d_L{}' \\ s_L{}' \\ b_L{}' \end{pmatrix} \\
&= \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} V_2^\dagger \gamma^\mu \mathrm{V} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \\
&= \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \gamma^\mu V_2^\dagger \mathrm{V} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}
\end{aligned}$$

define: $V_3 = V_2^\dagger \mathrm{V}$
$V_3 V_3^\dagger = (V_2^\dagger \mathrm{V})(V_2^\dagger \mathrm{V})^\dagger = V_2^\dagger \mathrm{V}\mathrm{V}^\dagger V_2 = 1$
$\rightarrow$ 1 matrix sufficient

- C: transforms particles into anti-particles
- P: inverts momentum

EM Interactions

$$
\begin{array}{llcl}
 & e^- & \rightarrow & \gamma e^- \\
 & L & \rightarrow & -1R \\
P & e^- & \rightarrow & \gamma e^- \\
 & R & \rightarrow & +1L \\
C & e^+ & \rightarrow & \gamma e^+ \\
 & L & \rightarrow & -1R \\
CP & e^+ & \rightarrow & \gamma e^+ \\
 & R & \rightarrow & +1L
\end{array}
$$

EW Interactions

$$
\begin{array}{llcl}
 & \mu^- & \rightarrow & \nu_{\mu_L} e^- \overline{\nu}_{e_L} \\
 & L & \rightarrow & LLR \\
P & \mu^- & \rightarrow & \nu_{\mu_L} e^- \overline{\nu}_{e_L} \\
 & R & \rightarrow & RRL \\
C & \mu^+ & \rightarrow & \overline{\nu}_{\mu_L} e^+ \nu_{e_L} \\
 & L & \rightarrow & LLR \\
CP & \mu^+ & \rightarrow & \overline{\nu}_{\mu_L} e^+ \nu_{e_L} \\
 & R & \rightarrow & RRL
\end{array}
$$

CPT always conserved

Neutral Kaons

$$\begin{aligned} \mathrm{K}^\circ &= |\bar{\mathrm{s}}\mathrm{d}\rangle \\ \overline{\mathrm{K}}^\circ &= -|\mathrm{s}\bar{\mathrm{d}}\rangle \end{aligned}$$

-: strong Isospin anti-particle

Charge Conjugation

$$\begin{aligned} C\mathrm{K}^\circ &= C(|\bar{\mathrm{s}}\mathrm{d}\rangle) \\ &= |\mathrm{s}\bar{\mathrm{d}}\rangle \\ &= -\overline{\mathrm{K}}^\circ \\ C\overline{\mathrm{K}}^\circ &= -|\bar{\mathrm{s}}\mathrm{d}\rangle \\ &= -\mathrm{K}^\circ \end{aligned}$$

not Eigenstates of C

Parity

- $(-1)^\ell$ from $Y_{\ell m}(\pi - \theta, \pi + \phi) = (-1)^\ell Y(\theta, \phi)$
- multiplicative: $P(p_1 p_2) = P(p_1) \cdot P(p_2)$
- Spinor: $\gamma^0\psi$ (DIRAC equation)
  - $\gamma^0 u(\mathbf{p}') = u(\mathbf{p})$
  - $\gamma^0 v(\mathbf{p}') = -v(\mathbf{p})$
- relative (-1) between particle and anti-particle

CP

$$
\begin{aligned}
CP\mathrm{K}^\circ &= (-1)\cdot(-1)^\ell(-\overline{\mathrm{K}}^\circ) \\
&= \overline{\mathrm{K}}^\circ \\
CP\overline{\mathrm{K}}^\circ &= (-1)\cdot(-1)^\ell(-\mathrm{K}^\circ) \\
&= \mathrm{K}^\circ
\end{aligned}
$$

$$J^P(\mathrm{K}^\circ) = J^P(\overline{\mathrm{K}}^\circ) = 0^-$$

CP Eigenstates

$$
\begin{aligned}
(+1):\quad K_1 &= \tfrac{1}{\sqrt{2}}(\mathrm{K}^\circ + \overline{\mathrm{K}}^\circ) \\
(-1):\quad K_2 &= \tfrac{1}{\sqrt{2}}(\mathrm{K}^\circ - \overline{\mathrm{K}}^\circ)
\end{aligned}
$$

strong prod, weak decay

$\pi^+\pi^-$

$$
\begin{aligned}
C(\pi^+\pi^-) &= \pi^-\pi^+ \\
P(\pi^+\pi^-) &= P(\pi^-)P(\pi^+) \\
&= 1 \\
CP(\pi^+\pi^-) &= 1
\end{aligned}
$$

$\pi^+\pi^-\pi^0$

$$
\begin{aligned}
C(\pi^0) &= C(\gamma)^2 = 1 \\
P(\pi^0) &= -1 \\
CP(\pi^+\pi^-\pi^0) &= CP(\pi^+\pi^-)\cdot \\
&\quad\; CP(\pi^0) \\
&= -1
\end{aligned}
$$

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Lifetimes

Kaon mass: 494MeV

$$\begin{aligned} K_1 &\rightarrow \pi^+\pi^- \\ \tau_S &= 0.9 \cdot 10^{-10} s \\ K_2 &\rightarrow \pi^+\pi^-\pi^0 \\ \tau_L &= 5.2 \cdot 10^{-8} s \end{aligned}$$

phase space:

$$\begin{aligned} m(\pi^+\pi^-) &\approx 280 MeV \\ m(\pi^+\pi^-\pi^0) &\approx 420 MeV \end{aligned}$$

K_2 was initially “overlooked”

Time dependence

Decay is described by weak Eigenstates with a well-defined lifetime:

$$\begin{aligned} |K_1(t)\rangle &= |K_1(0)\rangle \exp^{-iM_S t} \exp^{-\Gamma_S t/2} \\ |K_2(t)\rangle &= |K_2(0)\rangle \exp^{-iM_L t} \exp^{-\Gamma_L t/2} \end{aligned}$$

strong as f(weak):

$$\begin{aligned} \mathrm{K}^\circ &= \frac{1}{\sqrt{2}}(|K_1\rangle + |K_2\rangle) \\ \overline{\mathrm{K}}^\circ &= \frac{1}{\sqrt{2}}(|K_1\rangle - |K_2\rangle) \end{aligned}$$

Oscillation

$A_{\overline{K}^\circ K^\circ}(t)$: amplitude to produce at $t = 0$ a K° and find at t a $\overline{K}^\circ$

$$\begin{aligned}
A_{\overline{K}^\circ K^\circ}(t) &= \langle \overline{K}^\circ(t) | K^\circ(t=0) \rangle \\
&= \tfrac{1}{2}(\langle K_1(t)| - \langle K_2(t)|)(|K_1(t=0)\rangle + |K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\langle K_1(t)|K_1(t=0)\rangle - \langle K_2(t)|K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\exp^{-iM_S t} \exp^{-\Gamma_S t/2} - \exp^{-iM_L t} \exp^{-\Gamma_L t/2}) \\
P_{\overline{K}^\circ K^\circ}(t) &= A_{\overline{K}^\circ K^\circ}(t) A^\star_{\overline{K}^\circ K^\circ}(t) \\
&= \tfrac{1}{4}(\exp^{-\Gamma_S t} + \exp^{-\Gamma_L t} - 2\cos(\Delta M t) \exp^{-(\Gamma_L + \Gamma_S)t/2})
\end{aligned}$$

- $\Gamma_S \gg \Gamma_L$
- oscillation with frequency $\Delta M = M_L - M_S$

- $A_{\overline{\mathrm{K}}^\circ \mathrm{K}^\circ}(t)$: $t = 0$ a K° and at t a $\overline{\mathrm{K}}^\circ$
- $A_{\mathrm{K}^\circ \overline{\mathrm{K}}^\circ}(t)$: $t = 0$ a $\overline{\mathrm{K}}^\circ$ and at t a K°

$$\begin{aligned}
A_{\overline{\mathrm{K}}^\circ \mathrm{K}^\circ}(t) &= \tfrac{1}{2}(\langle K_1(t)| - \langle K_2(t)|)(|K_1(t=0)\rangle + |K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\langle K_1(t)|K_1(t=0)\rangle - \langle K_2(t)|K_2(t=0)\rangle) \\
A_{\mathrm{K}^\circ \overline{\mathrm{K}}^\circ}(t) &= \tfrac{1}{2}(\langle K_1(t)| + \langle K_2(t)|)(|K_1(t=0)\rangle - |K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\langle K_1(t)|K_1(t=0)\rangle - \langle K_2(t)|K_2(t=0)\rangle) \\
&= A_{\overline{\mathrm{K}}^\circ \mathrm{K}^\circ}((t) \\
P_{\overline{\mathrm{K}}^\circ \mathrm{K}^\circ}(t) &= P_{\mathrm{K}^\circ \overline{\mathrm{K}}^\circ}(t) \\
P_{\mathrm{K}^\circ \overline{\mathrm{K}}^\circ}(t) &= \tfrac{1}{4}(\exp^{-\Gamma_S t} + \exp^{-\Gamma_L t} - 2\cos(\Delta M t)\exp^{-(\Gamma_L+\Gamma_S)t/2})
\end{aligned}$$

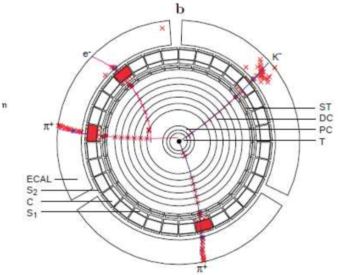
- $A_{\mathrm{K}^\circ\mathrm{K}^\circ}(t)$: $t=0$ a K° and at t a K°
- $A_{\overline{\mathrm{K}}^\circ\overline{\mathrm{K}}^\circ}(t)$: $t=0$ a $\overline{\mathrm{K}}^\circ$ and at t a $\overline{\mathrm{K}}^\circ$
- $A_{\Delta m}$ asymmetry used to express the experimental measurement. Overall normalisation is cancelled in the ratio.

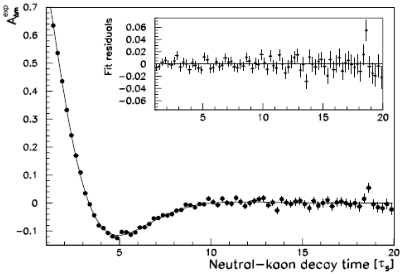
$$
\begin{aligned}
A_{\mathrm{K}^\circ\mathrm{K}^\circ}(t) &= \tfrac{1}{2}(\langle K_1(t)| + \langle K_2(t)|)(|K_1(t=0)\rangle + |K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\langle K_1(t)|K_1(t=0)\rangle + \langle K_2(t)|K_2(t=0)\rangle) \\
A_{\overline{\mathrm{K}}^\circ\overline{\mathrm{K}}^\circ}(t) &= \tfrac{1}{2}(\langle K_1(t)|K_1(t=0)\rangle + \langle K_2(t)|K_2(t=0)\rangle) \\
P_{\mathrm{K}^\circ\mathrm{K}^\circ}(t) &= P_{\overline{\mathrm{K}}^\circ\overline{\mathrm{K}}^\circ}(t) \\
P_{\mathrm{K}^\circ\mathrm{K}^\circ}(t) &= \tfrac{1}{4}(\exp^{-\Gamma_S t} + \exp^{-\Gamma_L t} + 2\cos(\Delta M t)\exp^{-(\Gamma_L+\Gamma_S)t/2}) \\
A_{\Delta m} &= \frac{(P_{\mathrm{K}^\circ\mathrm{K}^\circ}(t)+P_{\overline{\mathrm{K}}^\circ\overline{\mathrm{K}}^\circ}(t))-(P_{\overline{\mathrm{K}}^\circ\mathrm{K}^\circ}(t)+P_{\mathrm{K}^\circ\overline{\mathrm{K}}^\circ}(t))}{P_{\mathrm{K}^\circ\mathrm{K}^\circ}(t)+P_{\overline{\mathrm{K}}^\circ\overline{\mathrm{K}}^\circ}(t)+P_{\overline{\mathrm{K}}^\circ\mathrm{K}^\circ}(t)+P_{\mathrm{K}^\circ\overline{\mathrm{K}}^\circ}(t)} \\
&= 2\frac{\cos(\Delta M t)\exp^{-(\Gamma_L+\Gamma_S)t/2}}{\exp^{-\Gamma_S t} + \exp^{-\Gamma_L t}}
\end{aligned}
$$

A recent example (CPLear)

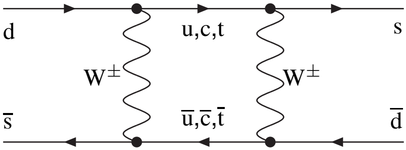
- 200MeV $\bar{p}$ to rest in Hydrogen
- need to tag the initial state: strange mesons in pairs
- need to tag the final state: sign of electron

$$\begin{aligned}
\bar{p}p &\rightarrow \mathrm{K}^-\pi^+\mathrm{K}^\circ \\
\bar{p}p &\rightarrow \mathrm{K}^+\pi^-\overline{\mathrm{K}}^\circ \\
\mathrm{K}^\circ &\rightarrow \pi^-\mathrm{e}^+\nu_{\mathrm{e_L}} \\
\overline{\mathrm{s}} &\rightarrow \overline{\mathrm{u}}\mathrm{e}^+\nu_{\mathrm{e_L}} \\
\overline{\mathrm{K}}^\circ &\rightarrow \pi^+\mathrm{e}^-\overline{\nu}_{\mathrm{e_L}} \\
\mathrm{s} &\rightarrow \mathrm{u}\mathrm{e}^-\overline{\nu}_{\mathrm{e_L}}
\end{aligned}$$

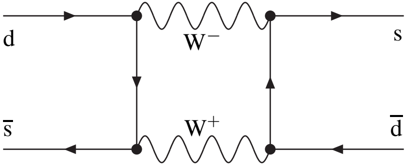




- at t=0: asymmetry zero
- need interference term!
- $A_{\Delta m} = 2\frac{\cos(\Delta Mt)\exp^{-(\Gamma_L+\Gamma_S)t/2}}{\exp^{-\Gamma_S t}+\exp^{-\Gamma_L t}}$
- $\Delta M \sim 3.5 \cdot 10^{-6} eV$

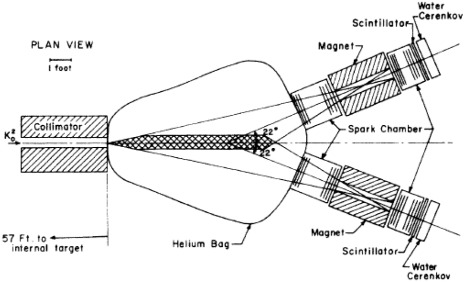


Follow fermion line: transition between generations inevitable!

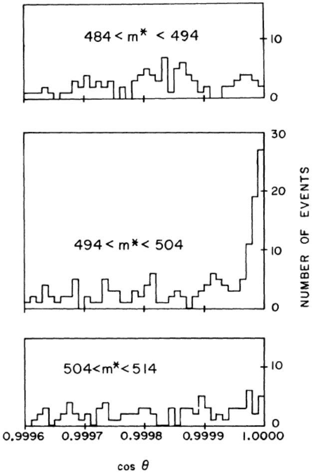


Need *CKM* non-diagonal: $\sim \sin^2 \theta_C$

All settled?



- BNL AGS 30GeV protons
- K_1 die out (can be regenerated)
- expect no $\pi^+\pi^-$ decays at the K° mass (theoretically)
- experimentally: combinatorics
- use angle between beam and reconstructed $\pi^+\pi^-$ system
- no peak expected



- PEAK!!!
- level: 10^{-3}
- *CP* must be violated!
- *CKM* has a complex phase

Kaon description

$$K_S = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_1\rangle + \epsilon|K_2\rangle)$$
$$K_L = \frac{1}{\sqrt{1+|\epsilon|^2}}(\epsilon|K_1\rangle + |K_2\rangle)$$

CP violation in mixing

$$|\epsilon| = \sqrt{\frac{\Gamma_L(\pi^+\pi^-)}{\Gamma_S(\pi^+\pi^-)}} = 2.268 \pm 0.023 \cdot 10^{-3}$$

CP violation in decay

$$Re(\frac{\epsilon'}{\epsilon}) = \frac{1}{6}(1 - \frac{\Gamma_L(\pi^0\pi^0)\Gamma_S(\pi^+\pi^-)}{\Gamma_S(\pi^0\pi^0)\Gamma_L(\pi^+\pi^-)})$$
$$(NA31) = 23 \pm 6.5 \cdot 10^{-4}$$
$$(FNAL) = 7.4 \pm 5.9 \cdot 10^{-4}$$
$$(FNAL) = 28 \pm 4.1 \cdot 10^{-4}$$
$$(NA48) = 18.5 \pm 7.3 \cdot 10^{-4}$$

- *CP* violation discovered
- good for our existence
- $\alpha_S(\mathrm{K}^\circ)$!

B-sector

- Flavour oscillation in all neutral systems
- $m_{B^\circ} \sim 5GeV \gg m_{K^\circ} \sim 0.5GeV$
- lifetime (tag)

Experiments

large production

- dedicated machine: e^+e^-
- or pp
- good PID

- BABAR@SLAC PEP-II
- BELLE@KEK-B
- asymmetric colliders
  - e^-: 9.1GeV
  - e^+: 3.4GeV
- Υ^{4s}:
  - resonance $b\bar{b}$
  - decay to $B_d^\circ \overline{B}_d^\circ$ and $B_s^\circ \overline{B}_s^\circ$
  - $250\mu m$ need great vertex detector

Back to *CKM*

- V complex
- $\mathrm{V}^\dagger \mathrm{V} = 1_3$: 9 equations
- 6 equations with complex $= 0$
- 2-coordinate plane: triangle
- α, β, γ

Unitary triangle

$$\begin{aligned}(\mathrm{V}^\dagger \mathrm{V})_{31} &= 0 \\ &= V^\star_{ub} V_{ud} + V^\star_{cb} V_{cd} + V^\star_{tb} V_{td}\end{aligned}$$

Measurements

- Δm_s
- Δm_d
- $\overline{\mathrm{B}}^\circ \to \pi^+\pi^-$
- $\overline{\mathrm{B}}^\circ \to J/\psi K_S$
- $B^+ \to DK^+$
- $B \to \tau\nu$
- overconstrained

- relationship angles-V
- all measurements in agreement
- no sign of BSM
- impressive progress in 10 years