Particle Physics
The Standard Model
Electroweak theory (I)

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ENS

Part VI

Electroweak theory (I)

History

Fermi

Left and Right

Lagrangien

Higgs

- charged leptons and photon
- quarks and gluon
- neutrinos
- $W^{\pm}, Z^{\circ}$
- H

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$$

$$\begin{matrix} u_R & c_R & t_R \\ d_R & s_R & b_R \\ e_R & \mu_R & \tau_R \end{matrix}$$

$$\begin{matrix} \gamma \\ g \\ W^{\pm}, Z^{\circ} \\ H \end{matrix}$$

History

- 1896 Henri Becquerel: β decay
- 1899 Ernest Rutherford: distinguishes α and β rays
- 1914 James Chadwick: the β decay has a continuous spectrum
- 1930 Wolfgang Pauli: postulates the neutrino (ballroom)
- 1933 Enrico Fermi: contact interaction
- 1953 Frederick Reines: $\overline{\nu}_{e_L} + p \rightarrow n + e^+$
- 1956 Lee, Yang, Wu, Garwin et al: Parity violation
- 1961 Glashow, Salam, Weinberg, Higgs, EBKGH
- 1973 Lagarrigue, Faissner: neutral currents (Z° t-channel)
- 1984 Rubbia, van der Meer : $W^\pm$, Z°
- 2012 discovery of the Higgs boson

$$
\begin{aligned}
n &\rightarrow p + \mathrm{e}^{-} + \overline{\nu}_{\mathrm{e_L}} \\
T_{fi} &\sim G(\bar{p}\gamma^{\mu} n)(\bar{e}\gamma_{\mu}\nu) \\
&\sim G(\bar{p}\gamma^{\mu} n)\frac{1}{q^2 - m^2}(\bar{e}\gamma_{\mu}\nu)
\end{aligned}
$$

- QED: $m^2 = 0 \rightarrow \frac{1}{q^2}$
- if $m^2 \gg q^2$ neglect q^2
- constant in momentum space $\rightarrow$ Dirac function in space-time: contact interaction
- $G \sim 10^{-5} GeV^{-2} \ll \alpha_{EM}$

Currents

$\bar{\psi}\psi$	*scalar*	*S*
$\bar{\psi}\gamma^{\mu}\psi$	*vector*	*V*
$\bar{\psi}\sigma^{\mu\nu}\psi$	*tensor*	*T*
$\bar{\psi}\gamma^{\mu}\gamma_5\psi$	*axial vector*	*A*
$\bar{\psi}\gamma_5\psi$	*pseudo scalar*	*PS*

QED: V
EW: $V - A$
$V - A$ violates parity (experiment)
$V - A$ quark/lepton level, not hadron level

Chirality

Chirality is the handed-ness of the particle:

$$\begin{aligned}\psi &= P_L\psi + P_R\psi \\ &= \psi_L + \psi_R\end{aligned}$$

Definitions

- Helicity: $\vec{\sigma} \cdot \vec{p}$
- $m = 0$: Helicity $=$ chirality
- $m = 0$: ψ and $\gamma_5\psi$ solve DIRAC

Weyl basis

$$\begin{aligned}\gamma_5 &= i\gamma^0\gamma^1\gamma^2\gamma^3 \\ \gamma_5^2 &= 1 \\ 0 &= \gamma_5\gamma^\mu + \gamma^\mu\gamma_5 \\ \gamma_5 &= \begin{pmatrix} -1_2 & 0 \\ 0 & 1_2 \end{pmatrix} \\ \gamma^0 &= \begin{pmatrix} 0 & 1_2 \\ 1_2 & 0 \end{pmatrix} \\ {\gamma^0}^\dagger &= \gamma^0 \\ {\gamma_5}^\dagger &= \gamma_5\end{aligned}$$

Left and Right chirality

$$
\begin{aligned}
P_L &= \tfrac{1}{2}(1-\gamma_5) \\
P_R &= \tfrac{1}{2}(1+\gamma_5) \\
P_L + P_R &= 1 \\
P_L^2 &= \tfrac{1}{2}(1-\gamma_5)\tfrac{1}{2}(1-\gamma_5) \\
&= \tfrac{1}{4}(1-\gamma_5)(1-\gamma_5) \\
&= \tfrac{1}{4}(1-\gamma_5-\gamma_5+\gamma_5^2) \\
&= \tfrac{1}{4}(1-\gamma_5-\gamma_5+1) \\
&= \tfrac{1}{2}(1-\gamma_5) \\
P_L P_R &= \tfrac{1}{2}(1-\gamma_5)\tfrac{1}{2}(1+\gamma_5) \\
&= \tfrac{1}{4}(1-\gamma_5)(1+\gamma_5) \\
&= \tfrac{1}{4}(1-\gamma_5+\gamma_5-\gamma_5^2) \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
P_L\psi &= P_L \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \\
&= \psi_L \\
\bar{\psi}P_L &= (P_L\gamma^0\psi)^\dagger \\
&= \psi_R^\dagger
\end{aligned}
$$

- $m = 0$ helicity conserved $\rightarrow \sigma \cdot \vec{p}$ good QN
- particle (**p**) $\rightarrow$ anti-particle $-$**p**: $\sigma \rightarrow -\sigma$
- ψ_R right-(left)handed (anti-)particle
- ψ_L left-(right)handed (anti-)particle

EM current

$$
\begin{aligned}
j^\mu &= -e\bar{\psi}\gamma^\mu\psi \\
&= -e\bar{\psi}(P_L + P_R)\gamma^\mu(P_L + P_R)\psi \\
&= -e\bar{\psi}P_L\gamma^\mu P_L\psi - e\bar{\psi}P_R\gamma^\mu P_R\psi - e\bar{\psi}P_R\gamma^\mu P_L\psi - e\bar{\psi}P_L\gamma^\mu P_R\psi \\
&= -e\bar{\psi}\gamma^\mu P_R P_L\psi - e\bar{\psi}\gamma^\mu P_L P_R\psi - e\bar{\psi}P_R\gamma^\mu P_L\psi - e\bar{\psi}P_L\gamma^\mu P_R\psi \\
&= -e\bar{\psi}P_R\gamma^\mu P_L\psi - e\bar{\psi}P_L\gamma^\mu P_R\psi
\end{aligned}
$$

Perfect symmetry under parity: $\vec{p} \rightarrow -\vec{p}$

- weak interaction: Left is not equal to Right
- use vector bosons
- ask for local gauge invariance
- remember that $U(1)_{EM}$ is QED and was extremely successful
- unify electromagnetic and weak interactions
- $SU(2) \times U(1)$
- $SU(2)$: three generators (gauge bosons)
- $U(1)$: one generators (gauge boson)
- $SU(2)$ vector bosons must be massive (Fermi-contact interaction)
- massive vector bosons lead to a non-renormalizable theory

The free Lagrangian ($\mathcal{L}_0$)

Remember QCD:

$$\begin{array}{ll} GaugeGroup & SU(3) \\ Gaugebosons & 8 \\ Lorentz-Vectors & G^a_\mu(\mathbf{x}) \\ Field-Tensor & G^a_{\mu\nu} = \partial_\mu G^a_\nu(\mathbf{x}) - \partial_\nu G^a_\mu(\mathbf{x}) - g_S f_{abc} G^b_\mu(\mathbf{x}) G^c_\nu(\mathbf{x}) \\ Structure & [\frac{\lambda_a}{2}, \frac{\lambda_b}{2}] = i f_{abc} \frac{\lambda_c}{2} \end{array}$$

$SU(2)_L$:

$$\begin{array}{ll} GaugeGroup & SU(2) \\ Gaugebosons & 3 \\ Lorentz-Vectors & W^a_\mu(\mathbf{x}) \\ Field-Tensor & W^a_{\mu\nu} = \partial_\mu W^a_\nu(\mathbf{x}) - \partial_\nu W^a_\mu(\mathbf{x}) - g_2 \epsilon_{abc} W^b_\mu(\mathbf{x}) W^c_\nu(\mathbf{x}) \\ Structure & [\frac{T_a}{2}, \frac{T_b}{2}] = i \epsilon_{abc} \frac{T_c}{2} \\ (T_a)_{3\times 3} & \begin{pmatrix} \sigma_a & 0 \\ 0 & 0 \end{pmatrix} \end{array}$$

$SU(2)_L$:

$GaugeGroup$	$SU(2)$
$Gaugebosons$	3
$Lorentz - Vectors$	$W^a_\mu(\mathbf{x})$
$Field - Tensor$	$W^a_{\mu\nu} = \partial_\mu W^a_\nu(\mathbf{x}) - \partial_\nu W^a_\mu(\mathbf{x}) - g\epsilon_{abc} W^b_\mu(\mathbf{x}) W^c_\nu(\mathbf{x})$
$Structure$	$[\frac{T_a}{2}, \frac{T_b}{2}] = i\epsilon_{abc}\frac{T_c}{2}$

$U(1)_Y$ Y weak hypercharge:

$GaugeGroup$	$U(1)$
$Gaugeboson$	1
$Lorentz - Vector$	$B_\mu(\mathbf{x})$
$Field - Tensor$	$B_{\mu\nu} = \partial_\mu B_\nu(\mathbf{x}) - \partial_\nu B_\mu(\mathbf{x})$

Free Lagrangian Gauge Fields $SU(2)_L \times U(1)_Y$

$$\begin{aligned} \mathcal{L}_0 &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu\, a} \\ &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} Tr(W_{\mu\nu} W^{\mu\nu}) \\ W_{\mu\nu} &= W^a_{\mu\nu} \frac{T_a}{2} \end{aligned}$$

Organize the Dirac Fields

$$\begin{aligned} \mathrm{e_L}(\mathbf{x}) &= P_L \mathrm{e}(\mathbf{x}) \\ \overline{\mathrm{e_L}}(\mathbf{x}) &= (\gamma^0 P_L \mathrm{e}(\mathbf{x}))^\dagger \\ \ell(\mathbf{x}) &= \begin{pmatrix} \nu_{\mathrm{e_L}}(\mathbf{x}) \\ \mathrm{e_L}(\mathbf{x}) \\ \mathrm{e_R}(\mathbf{x}) \end{pmatrix} \end{aligned}$$

No right-handed neutrinos

Define the weak Hypercharge

hypercharge left $\neq$ right:

$$Y = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & y_R \end{pmatrix}$$

$SU(2) \times U(1)$: y_R to be chosen later.....

Free Lagrangian Dirac Fields

$$\begin{aligned}\mathcal{L}_0 &= \overline{\nu_{e_L}}(\mathbf{x}) i\gamma^\mu \partial_\mu \nu_{e_L}(\mathbf{x}) \\ &+ \overline{e_L}(\mathbf{x}) i\gamma^\mu \partial_\mu e_L(\mathbf{x}) \\ &+ \overline{e_R}(\mathbf{x}) i\gamma^\mu \partial_\mu e_R(\mathbf{x}) \\ &= \bar{\ell}(\mathbf{x}) i\gamma^\mu \partial_\mu \ell(\mathbf{x})\end{aligned}$$

Minimal Substitution

$$\partial_\mu \quad \rightarrow \partial_\mu + ig_2 W^a_\mu \frac{T_a}{2} + ig_1 B_\mu \frac{Y}{2}$$

Interaction Lagrangian

$$\mathcal{L}' \quad = \quad -\bar{\ell}\gamma^\mu (g_2 W^a_\mu \frac{T_a}{2} + g_1 B_\mu \frac{Y}{2})\ell$$

- use form of Pauli matrices and $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$

Investigate the Interaction

$$\begin{aligned}
\mathcal{L}' &= -\bar{\ell}\gamma^\mu(g_2 W_\mu^a \tfrac{T_a}{2} + g_1 B_\mu \tfrac{Y}{2})\ell \\
&= -\bar{\ell}\gamma^\mu[g_2(W_\mu^1 \tfrac{T_1}{2} + W_\mu^2 \tfrac{T_2}{2} + W_\mu^3 \tfrac{T_3}{2}) + g_1 B_\mu \tfrac{Y}{2}]\ell \\
&= -g_2(\overline{\nu_{\mathrm{e_L}}}, \overline{\mathrm{e_L}})\gamma^\mu(W_\mu^1 \tfrac{\tau_1}{2} + W_\mu^2 \tfrac{\tau_2}{2} + W_\mu^3 \tfrac{\tau_3}{2})\begin{pmatrix}\nu_{\mathrm{e_L}} \\ \mathrm{e_L}\end{pmatrix} \\
&\quad + \tfrac{1}{2}g_1\overline{\nu_{\mathrm{e_L}}}\gamma^\mu B_\mu \nu_{\mathrm{e_L}} + \tfrac{1}{2}g_1\overline{\mathrm{e_L}}\gamma^\mu B_\mu \mathrm{e_L} - \tfrac{y_R}{2}g_1\overline{\mathrm{e_R}}\gamma^\mu B_\mu \mathrm{e_R} \\
&= -\tfrac{g_2}{\sqrt{2}}(W_\mu^+\overline{\nu_{\mathrm{e_L}}}\gamma^\mu \mathrm{e_L} + W_\mu^-\overline{\mathrm{e_L}}\gamma^\mu \nu_{\mathrm{e_L}}) \\
&\quad - \tfrac{1}{2}(g_2 W_\mu^3 - g_1 B_\mu)\overline{\nu_{\mathrm{e_L}}}\gamma^\mu \nu_{\mathrm{e_L}} \\
&\quad + \tfrac{1}{2}(g_2 W_\mu^3 + g_1 B_\mu)\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \tfrac{y_R}{2}g_1 B_\mu\overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R}
\end{aligned}$$

Identify the gauge bosons

- charged bosons: $W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(W^1_{\mu} \mp iW^2_{\mu})$
- neutral boson: $Z_{\mu} = \frac{1}{\sqrt{g_1^2+g_2^2}}(g_2 W^3_{\mu} - g_1 B_{\mu})$
- neutral boson: $A_{\mu} = \frac{1}{\sqrt{g_1^2+g_2^2}}(g_1 W^3_{\mu} + g_2 B_{\mu})$
- A_{μ} and Z_{μ} are orthogonal
- weak angle: $\sin\theta_W = \frac{g_1}{\sqrt{g_1^2+g_2^2}}$, $\cos\theta_W = \frac{g_2}{\sqrt{g_1^2+g_2^2}}$

$$\begin{aligned} B_{\mu} &= \frac{1}{\sqrt{g_1^2+g_2^2}}(g_2 A_{\mu} - g_1 Z_{\mu}) &= \cos\theta_W A_{\mu} - \sin\theta_W Z_{\mu} \\ W^3_{\mu} &= \frac{1}{\sqrt{g_1^2+g_2^2}}(g_1 A_{\mu} + g_2 Z_{\mu}) &= \sin\theta_W A_{\mu} + \cos\theta_W Z_{\mu} \end{aligned}$$

$$
\begin{aligned}
& -\tfrac{1}{2}(g_2 W^3_\mu - g_1 B_\mu)\overline{\nu_{\mathrm{e_L}}}\gamma^\mu \nu_{\mathrm{e_L}} + \tfrac{1}{2}(g_2 W^3_\mu + g_1 B_\mu)\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} \\
& -\tfrac{y_R}{2} g_1 B_\mu \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R} \\
= \quad & -\tfrac{1}{2}[g_2 \tfrac{1}{\sqrt{g_1^2+g_2^2}}(g_1 A_\mu + g_2 Z_\mu) - g_1 \tfrac{1}{\sqrt{g_1^2+g_2^2}}(g_2 A_\mu - g_1 Z_\mu)]\overline{\nu_{\mathrm{e_L}}}\gamma^\mu \nu_{\mathrm{e_L}} \\
& +\tfrac{1}{2}[g_2(\sin\theta_W A_\mu + \cos\theta_W Z_\mu) + g_1(\cos\theta_W A_\mu - \sin\theta_W Z_\mu)]\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} \\
& -\tfrac{y_R}{2} g_1(\cos\theta_W A_\mu - \sin\theta_W Z_\mu)\overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R} \\
= \quad & -\sqrt{g_1^2 + g_2^2} Z_\mu [\tfrac{1}{2}\overline{\nu_{\mathrm{e_L}}}\gamma^\mu \nu_{\mathrm{e_L}} - \tfrac{1}{2}\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} \\
& -\sin^2\theta_W(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} + \tfrac{y_R}{2}\overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})] \\
& -\tfrac{g_1 g_2}{\sqrt{g_1^2+g_2^2}} A_\mu(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} + \tfrac{y_R}{2}\overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})
\end{aligned}
$$

deduce $\frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}} = g_1 \cos\theta_W = g_2 \sin\theta_W = e$

deduce $y_R = -2$

$$\begin{aligned}\mathcal{L}' \quad = \quad & -\frac{e}{\sqrt{2}\sin\theta_W}\left(W^+_\mu \overline{\nu_{\mathrm{e_L}}}\gamma^\mu \mathrm{e_L} + W^-_\mu \overline{\mathrm{e_L}}\gamma^\mu \nu_{\mathrm{e_L}}\right) \\ & -\frac{e}{\sin\theta_W\cos\theta_W} Z_\mu \left[\tfrac{1}{2}\overline{\nu_{\mathrm{e_L}}}\gamma^\mu\nu_{\mathrm{e_L}} - \tfrac{1}{2}\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L}\right. \\ & \left. -\sin^2\theta_W(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})\right] \\ & -eA_\mu(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})\end{aligned}$$

- photon couples to charged particles only
- charged gauge bosons ensure transition between charged leptons and neutrinos
- a neutral gauge boson is predicted
- all gauge bosons are massless

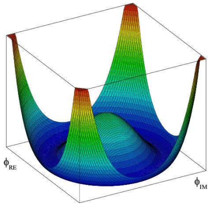
Introduce a complex scalar doublet:

$$\phi(\mathbf{x}) = \begin{pmatrix} \phi_1(\mathbf{x}) \\ \phi_2(\mathbf{x}) \end{pmatrix}, I^W = \frac{1}{2}$$

Free Lagrangian

$$\begin{aligned} \mathcal{L}_0 &= (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - V(\phi) \\ V(\phi) &= \kappa \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \end{aligned}$$

Theory must be stable: $\lambda > 0$
Minimum not at 0: $\kappa = -\mu^2 < 0$



The ground state is not unique:

$$\phi = \exp(i\frac{\tau_a}{2}\varphi_a) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

Choose $\varphi = 0 \rightarrow SU(2)$ symmetry is broken

Yukawa terms

$$\begin{aligned}
\mathcal{L}_Y &= -y_e \overline{\mathrm{e_R}} \phi^\dagger \begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix} \\
\textit{h.c.} & \quad -y_e (\overline{\nu_{\mathrm{e_L}}}, \overline{\mathrm{e_L}}) \phi \mathrm{e_R} \\
&= -y_e (\overline{\mathrm{e_R}} \phi_1^\dagger \nu_{\mathrm{e_L}} + \overline{\mathrm{e_R}} \phi_2^\dagger \mathrm{e_L}) \\
& \quad -y_e (\overline{\nu_{\mathrm{e_L}}} \phi_1 \mathrm{e_R} + \overline{\mathrm{e_L}} \phi_2 \mathrm{e_R})
\end{aligned}$$

Deduce the hypercharge:

$$\begin{aligned}
Q &= I_3^W + \frac{Y^W}{2} \\
0 &= -\frac{1}{2} + \frac{Y^W}{2} \\
y_H &= 1
\end{aligned}$$

Minimal Subsitution

$$\begin{aligned}
\partial_\mu \phi &\rightarrow \partial_\mu \phi + i g_2 W_\mu^a \frac{\tau_a}{2} \phi \\
& \quad + i g_1 B_\mu \frac{y_H}{2} \phi \\
\partial_\mu \phi^\dagger &\rightarrow \partial_\mu \phi^\dagger - \phi^\dagger i g_2 W_\mu^a \frac{\tau_a}{2} \\
& \quad - \phi^\dagger i g_1 B_\mu \frac{y_H}{2}
\end{aligned}$$

Calculate the interaction terms

$$\phi^\dagger (-i g_2 W_\mu^a \frac{\tau_a}{2} - i g_1 B_\mu \frac{y_H}{2})$$
$$(+i g_2 W_\mu^a \frac{\tau_a}{2} + i g_1 B_\mu \frac{y_H}{2}) \phi$$

$$\phi^T = (0, \sqrt{\frac{\mu^2}{2\lambda}}) = (0, \frac{v}{\sqrt{2}})$$

$$(0,\sqrt{\frac{\mu^2}{2\lambda}})(-ig_2W_\mu^a\frac{\tau_a}{2}-ig_1B_\mu\frac{y_H}{2})(+ig_2W_\mu^a\frac{\tau_a}{2}+ig_1B_\mu\frac{y_H}{2})\begin{pmatrix}0\\\sqrt{\frac{\mu^2}{2\lambda}}\end{pmatrix}$$

$$=(0,\sqrt{\frac{\mu^2}{2\lambda}})(g_2W_\mu^a\frac{\tau_a}{2}+g_1B_\mu\frac{y_H}{2})(g_2W_\mu^a\frac{\tau_a}{2}+g_1B_\mu\frac{y_H}{2})\begin{pmatrix}0\\\sqrt{\frac{\mu^2}{2\lambda}}\end{pmatrix}$$

$$=(0,\sqrt{\frac{\mu^2}{2\lambda}})\begin{pmatrix}\frac{2g_2g_1A_\mu+(g_2^2-g_1^2)Z_\mu}{2\sqrt{g_1^2+g_2^2}} & \frac{g_2}{\sqrt{2}}W_\mu^+\\ \frac{g_2}{\sqrt{2}}W_\mu^- & -\frac{\sqrt{g_1^2+g_2^2}}{2}Z_\mu\end{pmatrix}$$

$$\begin{pmatrix}\frac{2g_2g_1A_\mu+(g_2^2-g_1^2)Z_\mu}{2\sqrt{g_1^2+g_2^2}} & \frac{g_2}{\sqrt{2}}W_\mu^+\\ \frac{g_2}{\sqrt{2}}W_\mu^- & -\frac{\sqrt{g_1^2+g_2^2}}{2}Z_\mu\end{pmatrix}\begin{pmatrix}0\\\sqrt{\frac{\mu^2}{2\lambda}}\end{pmatrix}$$

$$=\frac{g_2^2v^2}{4}W_\mu^-W^{\mu+}+\frac{(g_1^2+g_2^2)v^2}{8}Z_\mu Z^\mu$$

The weak bosons have acquired a mass!

Charged lepton masses

$$\begin{aligned}\mathcal{L}_Y &= -y_e(\overline{e_R}\phi_1^\dagger \nu_{e_L} + \overline{e_R}\phi_2^\dagger e_L) - y_e(\overline{\nu_{e_L}}\phi_1 e_R + \overline{e_L}\phi_2 e_R) \\ &= -y_e(\overline{e_R}\frac{v}{\sqrt{2}}e_L) - y_e(\overline{e_L}\frac{v}{\sqrt{2}}e_R) \\ &= -y_e\frac{v}{\sqrt{2}}(\overline{e_R}e_L + \overline{e_L}e_R) \\ &= -y_e\frac{v}{\sqrt{2}}(\bar{e}e)\end{aligned}$$

Masses

$$\begin{aligned}m_e &= y_e\frac{v}{\sqrt{2}} \\ m^2_{W^\pm} &= \frac{g_2^2 v^2}{4} = \frac{e^2 v^2}{4\sin^2\theta_W} \\ m^2_{Z^\circ} &= \frac{(g_1^2+g_2^2)v^2}{4} = \frac{e^2 v^2}{4\sin^2\theta_W\cos^2\theta_W} \qquad \mathcal{L} \to EQM \\ \frac{m^2_{W^\pm}}{m^2_{Z^\circ}} &= \cos^2\theta_W\end{aligned}$$

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- Quantum Numbers weak Isospin $SU(2)_L$ of fermions
- weak hypercharge
- $Q = I_3^W + \frac{Y}{2}$
- numerical coincidence of $I_3^W = I_3^S$ for L

I^W	I_3^W	Y			
$\frac{1}{2}$	$\frac{1}{2}$, $-\frac{1}{2}$	$\frac{1}{3}$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$
$\frac{1}{2}$	$\frac{1}{2}$, $-\frac{1}{2}$	-1	$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$
0	0	$\frac{4}{3}$	u_R	c_R	t_R
0	0	$-\frac{2}{3}$	d_R	s_R	b_R
0	0	-2	e_R	μ_R	τ_R

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$$\overline{\mathrm{e}_\mathrm{L}} = (\gamma^0 P_L \mathrm{e})^\dagger = \mathrm{e}^\dagger P_L^\dagger \gamma^0 = \mathrm{e}^\dagger P_L \gamma^0 = \mathrm{e}^\dagger \gamma^0 P_R = \bar{\mathrm{e}} P_R$$

The interactions

$$\begin{aligned}
\mathcal{L}' &= -\frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ \overline{\nu_{\mathrm{e_L}}}\gamma^\mu \mathrm{e_L} + W_\mu^- \overline{\mathrm{e_L}}\gamma^\mu \nu_{\mathrm{e_L}}) \\
&\quad -\frac{e}{\sin\theta_W \cos\theta_W} Z_\mu [\tfrac{1}{2}\overline{\nu_{\mathrm{e_L}}}\gamma^\mu \nu_{\mathrm{e_L}} - \tfrac{1}{2}\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} \\
&\quad -\sin^2\theta_W(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})] \\
&\quad -eA_\mu(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})
\end{aligned}$$

Electromagnetic Current

$$\begin{aligned}
\mathcal{L}' &= -eA_\mu(-\bar{\mathrm{e}} P_R \gamma^\mu P_L \mathrm{e} - \bar{\mathrm{e}} P_L \gamma^\mu P_R \mathrm{e}) \\
&= -eA_\mu \bar{\mathrm{e}}\gamma^\mu Q \mathrm{e} = -eA_\mu \bar{\mathrm{e}}\gamma^\mu (I_3^W + \tfrac{Y}{2})\mathrm{e} \\
&= -eA_\mu j_{EM}^\mu
\end{aligned}$$

Charged Current

$$\begin{aligned}
\mathcal{L}' &= -\frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ \overline{\nu_{\mathrm{e_L}}}\gamma^\mu P_L \mathrm{e} + W_\mu^- \bar{\mathrm{e}} P_R \gamma^\mu \nu_{\mathrm{e_L}}) \\
&= -\frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ \overline{\nu_{\mathrm{e}}}\gamma^\mu P_L \mathrm{e} + W_\mu^- \bar{\mathrm{e}}\gamma^\mu P_L \nu_{\mathrm{e}})
\end{aligned}$$