Particle Physics
The Standard Model
Hadrons and Quantum Chromodynamics (I)

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Part IV

Hadrons and Quantum Chromodynamics (I)

- Hadrons
  - History
  - $SU(2)$ Isospin
  - $SU(3)$ Flavor
- QCD
  - Fragmentation

- Remember the particle zoo
- charged leptons and photon
- add u, d $SU(2)$-Isospin
- add s $SU(3)$-Flavour
- add gluon (g)
- add the other quarks

$$\begin{pmatrix} \mathrm{u_L} \\ \mathrm{d_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{c_L} \\ \mathrm{s_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{t_L} \\ \mathrm{b_L} \end{pmatrix}$$

$$\begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_\mathrm{L}} \\ \mu_\mathrm{L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_\mathrm{L}} \\ \tau_\mathrm{L} \end{pmatrix}$$

$$\begin{matrix} \mathrm{u_R} & \mathrm{c_R} & \mathrm{t_R} \\ \mathrm{d_R} & \mathrm{s_R} & \mathrm{b_R} \\ \mathrm{e_R} & \mu_\mathrm{R} & \tau_\mathrm{R} \end{matrix}$$

$$\begin{matrix} \gamma \\ g \\ \mathrm{W}^\pm, \mathrm{Z}^\circ \\ \mathrm{H} \end{matrix}$$

Definition

Quarks $\mathrm{u}, \mathrm{d}, \mathrm{c}, \mathrm{s}, \mathrm{t}, \mathrm{b}$
sometimes also called partons

Properties of the u

$$m_0 = 2.5 \pm 0.7\mathrm{MeV}(2GeV)$$

Properties of the d

$$m_0 = 5.0 \pm 0.8\mathrm{MeV}(2GeV)$$

Properties of the c

$$\begin{aligned} m_0 &= 1.27\mathrm{GeV} \\ \tau &= (1.040 \cdot 10^{-12})\mathrm{s} \quad c\bar{d} \\ c\tau &= 311.8\mu\mathrm{m} \end{aligned}$$

Properties of the s

$$\begin{aligned} m_0 &= 100 \pm 25\mathrm{MeV} \\ \tau &= (1.24 \cdot 10^{-8})\mathrm{s} \quad u\bar{s} \\ c\tau &= 3.7\mathrm{m} \quad 1st \end{aligned}$$

Properties of the t

$$\begin{aligned} m_0 &= 172.9 \pm 1.0\mathrm{GeV} \\ \tau &\sim 10^{-23}\ \mathrm{s} \\ c\tau &\sim 10^{-15}\ \mathrm{m} \end{aligned}$$

Properties of the b

$$\begin{aligned} m_0 &= 4.19 \pm 0.12\mathrm{GeV} \quad (\bar{M}S) \\ \tau &= (1.6 \cdot 10^{-12})\mathrm{s} \quad u\bar{b} \\ c\tau &= 492\mu\mathrm{m} \end{aligned}$$

History

- 1947: Discovery of the charged pion in cosmic rays
- 1947: V particles (kink plus nothing then Vertex with 2 tracks)
- 1950: neutral pion
- 1960s: lots of new hadronic particles
- attempt to order the zoo
- introduce additional quantum numbers, substructure
- makes only sense if predictions arise from these attempts to order (if number of parameters is equal to the number of particles to be described it is a waste of time)

$SU(2)$

- $SU(2)$: 2×2 matrix
- $UU^\dagger = 1_2$, $det(U) = 1$
- $U = 1 + i \sum_{a=1}^{3} \delta\phi_a \frac{\tau_a}{2}$ with $\tau_a = \sigma_a$

Pions: 140MeV, Spin-0

- $I = 1 \rightarrow \pm 1, 0$
- $I_3|\pi^+\rangle = |\pi^+\rangle$
- $I_3|\pi^-\rangle = -|\pi^-\rangle$
- Kemmer predicted a neutral particle:
- $I_3|\pi^0\rangle = 0|\pi^0\rangle$

Nucleons: 1GeV, Spin-$\frac{1}{2}$

- electron spin: $\pm\frac{1}{2}$
- new QN: Isospin I (behaves spin-like)
- $m_p \approx m_n$
- $I = \frac{1}{2}$
- $I_3|p\rangle = \frac{1}{2}|p\rangle$
- $I_3|n\rangle = -\frac{1}{2}|n\rangle$

Order with Spin and Isospin: 5 particles described with quantum number

Baryons

- System of three quarks
- $|p\rangle = |uud\rangle$
- $|n\rangle = |udd\rangle$

Mesons

- System of quark anti-quark
- $|\pi^+\rangle = -|u\bar{d}\rangle$
- $|\pi^0\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle)$
- $|\pi^-\rangle = |d\bar{u}\rangle$

Hypercharge

$$Q = I_3 + \frac{1}{2}Y$$

therefore:

$$\begin{aligned} Y &= 2(Q - I_3) \\ Y(u) &= 2(\tfrac{2}{3} - \tfrac{1}{2}) \\ &= \tfrac{1}{3} \\ Y(d) &= \tfrac{1}{3} \\ Y(\bar{u}) &= -Y(u) \\ Y(\bar{d}) &= -Y(d) \end{aligned}$$

for the anti-quarks both charge AND isospin change signs

Proof.

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

Charge conjugation:

$$\begin{pmatrix} \bar{\mathrm{u}}' \\ \bar{\mathrm{d}}' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \bar{\mathrm{u}} \\ \bar{\mathrm{d}} \end{pmatrix}$$

Respect Charge ordering (index 1 $\leftrightarrow$ 2):

$$\begin{pmatrix} \bar{\mathrm{d}}' \\ \bar{\mathrm{u}}' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \bar{\mathrm{d}} \\ \bar{\mathrm{u}} \end{pmatrix}$$

Rewrite to obtain the same rotation matrix as for particles:

$$\begin{pmatrix} -\bar{\mathrm{d}}' \\ \bar{\mathrm{u}}' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} -\bar{\mathrm{d}} \\ \bar{\mathrm{u}} \end{pmatrix}$$

□

$SU(2)$ Flavor

Singlet State

$$\frac{1}{\sqrt{2}}(\overline{\mathrm{d}}\mathrm{d} + \overline{\mathrm{u}}\mathrm{u})$$

Triplet State

$$-\overline{\mathrm{d}}\mathrm{u}$$

$$\frac{1}{\sqrt{2}}(-\overline{\mathrm{d}}\mathrm{d} + \overline{\mathrm{u}}\mathrm{u})$$

$$\overline{\mathrm{u}}\mathrm{d}$$

- irreducible representations: $2^* \times 2 = 1 \oplus 3$
- exchange: $\mathrm{u} \leftrightarrow \mathrm{d}$
  - symmetric singlet
  - antisymmetric triplet
- need 3 generators for $SU(2)$ symmetry: triplet

Something strange was observed

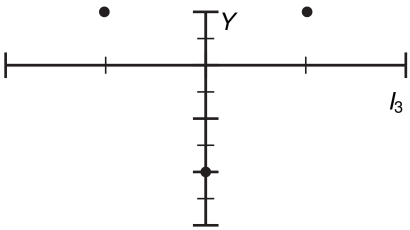
- 1953: production of V^0s in accelerators
- $\pi^- p \to \mathrm{K}^\circ \Lambda \to \pi^+ \pi^- p \pi^-$
- $\sigma \sim 1 mb \approx 10^{-31} m^2 \approx (10^{-15} m)^2 = (fm)^2$
- cross section of the order of the geometrical hadron radius
- $\tau \sim 10^{-10} s$
- or: strong interaction $\tau = \frac{1 fm}{3 \cdot 10^8 m/s} \approx 10^{-23} s$
- new QN: strangeness (conserved by strong interaction) $S(\mathrm{K}^\circ) = +1$, $S(\Lambda) = -1$
- modern formulation: introduce a new quark: s
- introduce a QN: S (strangeness)

- The hypercharge is redefined: $Y = S + B$
- Gell-Mann-Nishijima: $Q = I_3 + \frac{1}{2}Y$

B: Baryon number

quarks: $\frac{1}{3}$
anti-quarks: $-\frac{1}{3}$
Mesons (quark-anti-quark systems): 0
Baryons (3 quark system): 1

	I	I_3	Y	S	B	Q
u	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$
d	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$-\frac{1}{3}$
s	0	0	$-\frac{2}{3}$	-1	$\frac{1}{3}$	$-\frac{1}{3}$



$SU(3)$

- $|\mathrm{u}\rangle, |\mathrm{d}\rangle, |\mathrm{s}\rangle$
- $|\mathrm{u}\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, |\mathrm{d}\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},$
  $|\mathrm{s}\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
- $UU^\dagger = 1_3, det(U) = 1$
- $U = 1 + i\sum_{a=1}^{8} \delta\phi_a \frac{\lambda_a}{2}$
- $3 \times 3 \times 2 - 9 - 1 = 8$ generators

- Isospin $SU(2)$, hypercharge (a number) $U(1)$ gives $SU(2) \times U(1)$
- Gell-Mann-Ne'eman: $SU(3)$ can be decomposed into $SU(2) \times U(1)$

Gell-Mann Matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix}$$

- λ_1, λ_2, λ_3: Pauli matrices of $SU(2)$
- $\frac{1}{2}\lambda_3$: I_3
- $\frac{1}{\sqrt{3}}\lambda_8$: hypercharge
- λ_4, λ_5 (Pauli equivalent): u and s
- λ_6, λ_7 (Pauli equivalent): d and s

SU(3) Mesons

$\overline{\mathrm{u}}\overline{\mathrm{d}}\overline{\mathrm{s}}$ combined with uds

Singlet

$$\frac{1}{\sqrt{3}}(\overline{\mathrm{u}}\mathrm{u}+\overline{\mathrm{d}}\mathrm{d}+\overline{\mathrm{s}}\mathrm{s})$$

Oktuplet: eight-fold way

$$-\overline{\mathrm{d}}\mathrm{u}$$

$$\frac{1}{\sqrt{2}}(-\overline{\mathrm{d}}\mathrm{d}+\overline{\mathrm{u}}\mathrm{u})$$

$$\overline{\mathrm{u}}\mathrm{d}$$

$$\overline{\mathrm{s}}\mathrm{d}$$

$$-\overline{\mathrm{d}}\mathrm{s}$$

Oktuplet cont'd

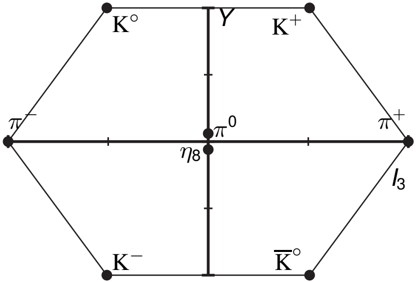
$$\overline{\mathrm{u}}\mathrm{s}$$

$$\overline{\mathrm{s}}\mathrm{u}$$

$$\frac{1}{\sqrt{6}}(\overline{\mathrm{u}}\mathrm{u}+\overline{\mathrm{d}}\mathrm{d}-2\overline{\mathrm{s}}\mathrm{s})$$

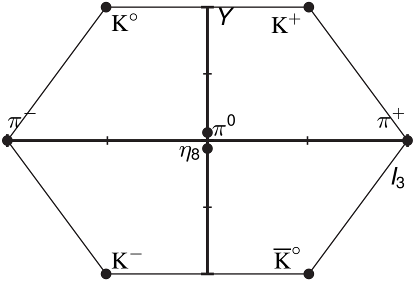
Multiplets: Mesons

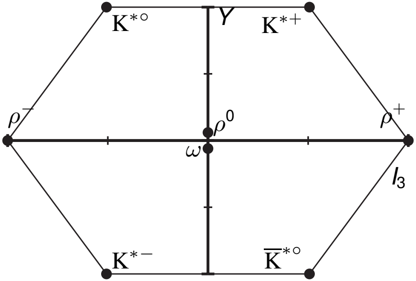
- $\eta_1 \sim \frac{1}{\sqrt{3}}(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} + \mathrm{s\bar{s}})$
- $\eta_8 \sim \frac{1}{\sqrt{6}}(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} - 2\mathrm{s\bar{s}})$



Ladder operators with Gell-Mann Matrices

$$
\begin{array}{lllllll}
I_\pm & = & \frac{1}{2}(\lambda_1 \pm i\lambda_2) & \Delta I_3 & = & \pm 1 & & \mathrm{d} \leftrightarrow \mathrm{u} \\
V_\pm & = & \frac{1}{2}(\lambda_4 \pm i\lambda_5) & \Delta I_3 & = & \pm\frac{1}{2} & \Delta Y = \pm 1 & \mathrm{s} \leftrightarrow \mathrm{u} \\
U_\pm & = & \frac{1}{2}(\lambda_6 \pm i\lambda_7) & \Delta I_3 & = & \mp\frac{1}{2} & \Delta Y = \pm 1 & \mathrm{s} \leftrightarrow \mathrm{d}
\end{array}
$$





2 meson octets

- same flavor content
- Pseudo-scalar meson spectrum
- Vector meson spectrum

SU(3) Baryons

Di-quarks

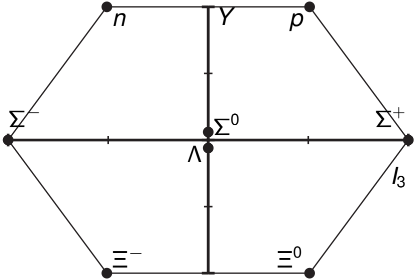
$$\mathrm{uu}$$
$$\mathrm{dd}$$
$$\mathrm{ss}$$
$$\frac{1}{\sqrt{2}}(\mathrm{ud}+\mathrm{du})$$
$$\frac{1}{\sqrt{2}}(\mathrm{us}+\mathrm{su})$$
$$\frac{1}{\sqrt{2}}(\mathrm{ds}+\mathrm{sd})$$
$$\frac{1}{\sqrt{2}}(\mathrm{ud}-\mathrm{du})$$
$$\frac{1}{\sqrt{2}}(\mathrm{us}-\mathrm{su})$$
$$\frac{1}{\sqrt{2}}(\mathrm{ds}-\mathrm{sd})$$

Decomposition

- 6 symmetric states and 3 anti-symmetric states
- $I_3^S(\mathrm{ud}-\mathrm{du}) = 0$
- $I_3^S(\mathrm{ud}+\mathrm{du}) = 0$
- $Y^S(\mathrm{ud}-\mathrm{du}) = \frac{2}{3} = Y^S(\bar{\mathrm{s}})$
- $3 \times 3 = 6 \oplus 3^*$
- $3 \times 3 \times 3 = (6 \oplus 3^*) \times 3 = 6 \times 3 \oplus 3^* \times 3$
- from the mesons we know: $3^* \times 3 = 1 \oplus 8$
- $6 \times 3 = 8 \oplus 10$

Multiplets: Baryons

- uds:
- $3 \times 3 \times 3 = 1 + 8 + 8 + 10$
- Λ and Σ°: uds, but isopin singlet versus triplet



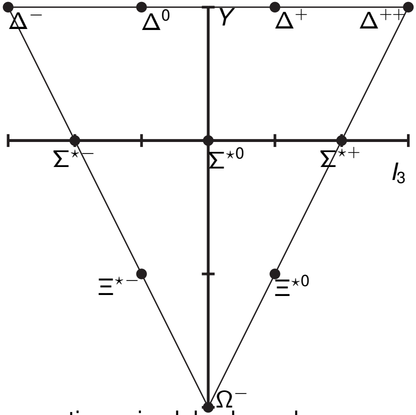
$$
\begin{aligned}
I_-|u\rangle &= |d\rangle \\
I_-|d\rangle &= 0 \\
I_-|p\rangle &= I_-|u\rangle|u\rangle|d\rangle \\
&= I_-(|u\rangle)|u\rangle|d\rangle + |u\rangle I_-(|u\rangle)|d\rangle + |u\rangle|u\rangle I_-|d\rangle \\
&\sim |d\rangle|u\rangle|d\rangle + |u\rangle|d\rangle|d\rangle
\end{aligned}
$$

Baryons

- $\Delta^{++} = |\mathrm{uuu}\rangle$
- $\Omega^{-} = |\mathrm{sss}\rangle$
- Ω^{-} predicted before discovery

Dekuplet

- symmetric flavor wave functions
- ladder operators keep wave functions symmetric
- 10 different states



$SU(3) - \mathit{Flavor}$ beyond order: relate cross sections via clebsch gordon coefficients

Gell-Mann Okubo

Physical state

- $m_{\Omega^-}/m_{\Delta^{++}} = 1672/1230$
- mass is a perturbation to $SU(3)_F$
- $m(\eta)\text{obs} = 548 MeV$
- $m(\eta')\text{obs} = 959 MeV$
- $m(\eta_8)\text{predicted} = 620 MeV$
- physical states do not have to follow $SU(3)_F$
- η and η_8 close in mass
- physical states: η, η' with $\cos\theta = 0.96$

η and η'

$$
\begin{aligned}
\langle u\bar{u}|M|\bar{u}u\rangle &= 2m_u \\
\eta_8 &= \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s) \\
m(\eta_8) &= \frac{1}{6}(\langle u\bar{u}|M|\bar{u}u\rangle \\
&\quad +\langle d\bar{d}|M|\bar{d}d\rangle \\
&\quad +4\langle s\bar{s}|M|\bar{s}s\rangle) \\
&\approx \frac{1}{3}(2m_u + 4m_s) \\
4m(K^+) &- m(\pi+) \\
&= 4(m_u + m_s) - 2m_u \\
&= 2m_u + 4m_s \\
&= 3m(\eta_8)
\end{aligned}
$$

$SU(6)$

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow \begin{pmatrix} u\uparrow \\ u\downarrow \\ d\uparrow \\ d\downarrow \\ s\uparrow \\ s\downarrow \end{pmatrix}$$

$SU(6)$

- describes *Spin* $\times$ *Flavor*
- symmetric subset: 56
- $S = \frac{1}{2}$ octet and $S = \frac{3}{2}$ decuplet
- all particles, why?

Spin for baryons

$$\begin{array}{l}
\uparrow\uparrow\uparrow \\
\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \\
\frac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow) \\
\downarrow\downarrow\downarrow \\
\frac{1}{\sqrt{6}}(2\downarrow\downarrow\uparrow - \uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow) \\
\frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \\
\frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow) \\
\frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)
\end{array}$$

- symmetric
- mixed symmetric

- EM: violates I, but preserves I_3: $\pi^0 \rightarrow \gamma\gamma$, $1 \rightarrow 0$, $0 \rightarrow 0$

charm, bottom, top

- What glue holds the hadrons together?
- Are quarks math or particles?
- photon carrier of EM-interactions
- gluon

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$\begin{pmatrix} \\ e_L \end{pmatrix} \begin{pmatrix} \\ \mu_L \end{pmatrix} \begin{pmatrix} \\ \tau_L \end{pmatrix}$$

$$\begin{matrix} u_R & c_R & t_R \\ d_R & s_R & b_R \\ e_R & \mu_R & \tau_R \end{matrix}$$

$$\gamma$$
$$g$$

QCD

1972,1973 Gell-Mann, Fritzsch, Leutwyler : $SU(3) - Color$

$$\mathcal{L}_0 = \bar{\psi}(\mathbf{x})(i\gamma^\mu \partial_\mu - m)\psi(\mathbf{x})$$

extend to 6 quarks (u, d, c, s, t, b):

$$\mathcal{L}_0 = \sum_{j=1}^{6} \bar{\psi}^j(\mathbf{x})(i\gamma^\mu \partial_\mu - m)\psi^j(\mathbf{x})$$

extend to three colors

$$\mathcal{L}_0 = \sum_{j=1}^{6} \sum_{c} \bar{\psi}^j_c(\mathbf{x})(i\gamma^\mu \partial_\mu - m)\psi^j_c(\mathbf{x})$$

define:

$$\mathbf{q}^j = \begin{pmatrix} \psi^j_R(\mathbf{x}) \\ \psi^j_G(\mathbf{x}) \\ \psi^j_B(\mathbf{x}) \end{pmatrix}$$

leads to:

$$\mathcal{L}_0 = \sum_{j=1}^{6} \bar{\mathbf{q}}^j(i\gamma^\mu \partial_\mu - m)\mathbf{q}^j$$

invariant under a global transformation U (color-index) with $UU^\dagger = 1_3$ and $det(U) = 1$

The free Lagrangian ($\mathcal{L}_0$)

Remember QED:

$$\begin{array}{ll} GaugeGroup & U(1) \\ Gaugebosons & 1 \\ Lorentz - Vector & A_\mu(\mathbf{x}) \\ Field - Tensor & F_{\mu\nu} = \partial_\mu A_\nu(\mathbf{x}) - \partial_\nu A_\mu(\mathbf{x}) \end{array}$$

QCD:

$$\begin{array}{ll} GaugeGroup & SU(3) \\ Gaugebosons & 8 \\ Lorentz - Vectors & G^a_\mu(\mathbf{x}) \\ Field - Tensor & G^a_{\mu\nu} = \partial_\mu G^a_\nu(\mathbf{x}) - \partial_\nu G^a_\mu(\mathbf{x}) - g_S f_{abc} G^b_\mu(\mathbf{x}) G^c_\nu(\mathbf{x}) \end{array}$$

Structure constants of $SU(3)$ (totally anti-symmetric):

$$[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}] = i f_{abc} \frac{\lambda_c}{2}$$

The additional term is characteristic of a non-Abelian theory

Define a hermitian matrix with the gluon Lorentz-Vectors (λ_a are hermitian, $G^a_\mu(\mathbf{x})$ is real):

$$\mathbf{G}_\mu(\mathbf{x}) = G^a_\mu(\mathbf{x})\frac{\lambda_a}{2} = (G^a_\mu(\mathbf{x}))^\dagger \frac{\lambda_a^\dagger}{2} = \mathbf{G}^\dagger_\mu(\mathbf{x})$$

Define a Field Tensor:

$$\begin{aligned}
& \mathbf{G}_{\mu\nu}(\mathbf{x}) \\
= \quad & \partial_\mu \mathbf{G}_\nu(\mathbf{x}) - \partial_\nu \mathbf{G}_\mu(\mathbf{x}) + ig_S[\mathbf{G}_\mu(\mathbf{x}), \mathbf{G}_\nu(\mathbf{x})] \\
= \quad & \partial_\mu(G^a_\nu(\mathbf{x})\tfrac{\lambda_a}{2}) - \partial_\nu(G^a_\mu(\mathbf{x})\tfrac{\lambda_a}{2}) + ig_S[G^b_\mu(\mathbf{x})\tfrac{\lambda_b}{2}, G^c_\nu(\mathbf{x})\tfrac{\lambda_c}{2}] \\
= \quad & (\partial_\mu G^a_\nu(\mathbf{x}) - \partial_\nu G^a_\mu(\mathbf{x}))\tfrac{\lambda_a}{2} + ig_S G^b_\mu(\mathbf{x}) G^c_\nu(\mathbf{x})[\tfrac{\lambda_b}{2}, \tfrac{\lambda_c}{2}] \\
= \quad & (\partial_\mu G^a_\nu(\mathbf{x}) - \partial_\nu G^a_\mu(\mathbf{x}))\tfrac{\lambda_a}{2} - g_S f_{bca} G^b_\mu(\mathbf{x}) G^c_\nu(\mathbf{x})\tfrac{\lambda_a}{2} \\
= \quad & (\partial_\mu G^a_\nu(\mathbf{x}) - \partial_\nu G^a_\mu(\mathbf{x}))\tfrac{\lambda_a}{2} - g_S f_{abc} G^b_\mu(\mathbf{x}) G^c_\nu(\mathbf{x})\tfrac{\lambda_a}{2} \\
= \quad & G^a_{\mu\nu}(\mathbf{x})\tfrac{\lambda_a}{2}
\end{aligned}$$

Free Lagrangian:

$$
\begin{aligned}
\mathcal{L}_0 &= -\frac{1}{2} Tr(\mathbf{G}_{\mu\nu}(\mathbf{x})\mathbf{G}^{\mu\nu}(\mathbf{x})) \\
&= -\frac{1}{2} Tr((\sum_a G^a_{\mu\nu}(\mathbf{x})\frac{\lambda_a}{2})(\sum_b G^{\mu\nu b}(\mathbf{x})\frac{\lambda_b}{2})) \\
&= -\frac{1}{2} Tr(\sum_{a,b} G^a_{\mu\nu}(\mathbf{x})G^{\mu\nu b}(\mathbf{x})\frac{\lambda_a}{2}\frac{\lambda_b}{2}) \\
&= -\frac{1}{2} \sum_{a,b} G^a_{\mu\nu}(\mathbf{x})G^{\mu\nu b}(\mathbf{x})\, Tr(\frac{\lambda_a}{2}\frac{\lambda_b}{2}) \\
&= -\frac{1}{2} \sum_{a,b} G^a_{\mu\nu}(\mathbf{x})G^{\mu\nu b}(\mathbf{x})\frac{1}{4} Tr(\lambda_a\lambda_b) \\
&= -\frac{1}{2} \sum_{a,b} G^a_{\mu\nu}(\mathbf{x})G^{\mu\nu b}(\mathbf{x})\frac{1}{4} 2\delta_{ab} \\
&= -\frac{1}{4} G^a_{\mu\nu}(\mathbf{x})G^{\mu\nu a}(\mathbf{x})
\end{aligned}
$$

Minimal substitution

$$\partial_\lambda \to \mathbf{D}_\lambda = \partial_\lambda + ig_S\mathbf{G}_\lambda(\mathbf{x}) + iqeA_\lambda(\mathbf{x})$$

where q is the charge of the quark and e is the elementary charge (> 0).
$q = -1$ for the electron.

$$\begin{aligned}\mathcal{L} &= -\tfrac{1}{2}Tr(\mathbf{G}_{\mu\nu}(\mathbf{x})\mathbf{G}^{\mu\nu}(\mathbf{x})) + \textstyle\sum_{j=1}^{6}\bar{\mathbf{q}}^j(i\gamma^\lambda D_\lambda - m_j))\mathbf{q}^j \\ &= -\tfrac{1}{4}G^a_{\mu\nu}(\mathbf{x})G^{\mu\nu a}(\mathbf{x}) \\ &+ \textstyle\sum_{j=1}^{6}\bar{\mathbf{q}}^j(i\gamma^\lambda(\partial_\lambda + ig_S G^a_\lambda \tfrac{\lambda_a}{2} + iqeA_\lambda(\mathbf{x}) - m_j))\mathbf{q}^j\end{aligned}$$

Lagrangian is invariant under local transformations $SU(3)_C$ (not shown) and $U(1)_{EM}$

$$\mathbf{G}_\mu(\mathbf{x}) \to U(\mathbf{x})\mathbf{G}_\mu(\mathbf{x})U^\dagger(\mathbf{x}) - \frac{i}{g_S}U(\mathbf{x})\partial_\mu U^\dagger(\mathbf{x})$$

The Ω^- puzzle

- $\Omega^- = |sss\rangle$
- $J(\Omega^-) = \frac{3}{2}$
- $\Omega^- = |s\uparrow s\uparrow s\uparrow\rangle$
- violates Pauli: fermions are anti-symmetric
- deduce hidden quantum number: QCD

The Ω^- solution

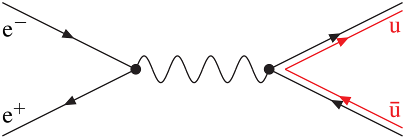
- $\Omega^- = \epsilon_{ijk} s_i s_j s_k$

Color

- $|u\rangle \to |u\rangle, |u\rangle, |u\rangle$
- $\langle u|u\rangle = \langle u|u\rangle = 0$

ENS

Final state

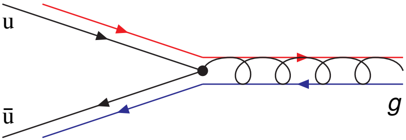


- $e^+e^- \to \gamma \to u\bar{u}$
- if color is not measured: sum of color
- $\sigma \sim N_C = 3$
- $\sigma \sim N_C \cdot q^2$

Initial state

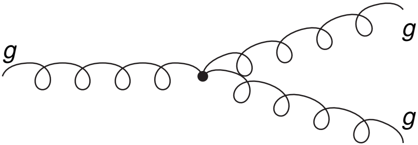
- $u\bar{u} \to \gamma \to e^+e^-$
- if color is not measured: average
- $\langle u|u\rangle = 1$ $\langle u|u\rangle = 1$ $\langle u|u\rangle = 1$
  $\langle u|u\rangle = 0$ $\langle u|u\rangle = 0$ $\langle u|u\rangle = 0$
  $\langle u|u\rangle = 0$ $\langle u|u\rangle = 0$ $\langle u|u\rangle = 0$
- $\sigma \sim \frac{3}{9} = \frac{1}{3}$

qqg

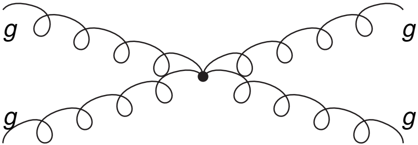


- gluon carries color and anti-color charge
- $g\mathrm{q}\bar{\mathrm{q}}$ vertex: $\sim g_S$ ($\alpha_S = \frac{g_S^2}{4\pi}$) electric charge irrelevant!

TGV and QGV



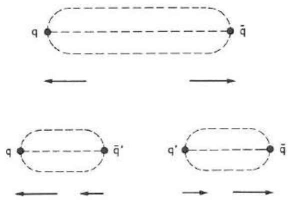
Non-abelian theory: triple gluon vertex $\sim g_S$



four gluon vertex $\sim g_S^2$

Fragmentation

- connection between hadrons and quarks?
- no colored particles observed
- Lund string fragmentation ($V \sim kr$)



- $\sqrt{s} = 1\,\mathrm{GeV}$
- $|\mathrm{K}^+\rangle = |\mathrm{u}\bar{\mathrm{s}}\rangle$
- $m_{\mathrm{K}^+} = 0.494\mathrm{GeV}$
- $\mathrm{e}^+\mathrm{e}^- \rightarrow \mathrm{s}\bar{\mathrm{s}} \rightarrow \mathrm{K}^+\mathrm{K}^-$
- more difficult at $\sqrt{s} \gg 2m$