# Particle Physics
# The Standard Model
## Quantum Electrodynamics (II)

Frédéric Machefert

frederic@cern.ch
Laboratoire de l'accélérateur linéaire (CNRS)

Cours de l'École Normale Supérieure
24, rue Lhomond, Paris

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# Part III

## Quantum Electrodynamics (II)

Muon pair production

Anomalous magnetic moment

- Remember the particle zoo
- $\gamma$ and $\mathrm{e}$
- today: add $\mu$ and $\tau$

### Definition

Charged Leptons: $\mathrm{e}$, $\mu$, $\tau$
Leptons: charged leptons plus neutrinos
Jargon: leptons as charged leptons

$$\begin{pmatrix} \mathrm{u_L} \\ \mathrm{d_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{c_L} \\ \mathrm{s_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{t_L} \\ \mathrm{b_L} \end{pmatrix}$$

$$\begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_\mathrm{L}} \\ \mu_\mathrm{L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_\mathrm{L}} \\ \tau_\mathrm{L} \end{pmatrix}$$

$$\begin{matrix} \mathrm{u_R} & \mathrm{c_R} & \mathrm{t_R} \\ \mathrm{d_R} & \mathrm{s_R} & \mathrm{b_R} \\ \mathrm{e_R} & \mu_\mathrm{R} & \tau_\mathrm{R} \end{matrix}$$

$$\begin{matrix} \gamma \\ g \\ \mathrm{W}^\pm, \mathrm{Z}^\circ \\ \mathrm{H} \end{matrix}$$

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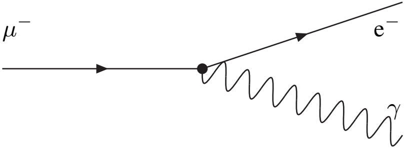
## Properties of the $\mu$

$$
\begin{array}{llll}
m_0 & = & 0.105\,\mathrm{GeV} & \mu^+e^- \\
\tau & = & (2.197\cdot 10^{-6})\,\mathrm{s} & \mathrm{PSI} \\
c\tau & = & 659\,\mathrm{m} &
\end{array}
$$



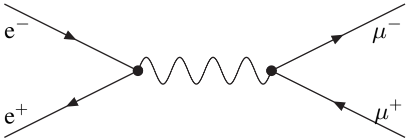

$$
\begin{array}{lll}
\mathcal{B}(\mu \to e\gamma) & < & 5.7\cdot 10^{-13} \\
\mathcal{B}(\tau \to e\gamma) & < & 3.3\cdot 10^{-8} \\
\mathcal{B}(\tau \to \mu\gamma) & < & 4.4\cdot 10^{-8} \\
CL & = & 90\%
\end{array}
$$

## Properties of the $\tau$

$$
\begin{array}{llll}
m_0 & = & 1.777\,\mathrm{GeV} & e^+e^- \\
\tau & = & (2.906\cdot 10^{-13})\,\mathrm{s} & e^+e^- \\
c\tau & = & 87\,\mu\mathrm{m} &
\end{array}
$$

## Lepton numbers (additive QNs)

| | $L_e$ | $L_\mu$ | $L_\tau$ |
|---|---|---|---|
| $e^-$ | 1 | 0 | 0 |
| $e^+$ | −1 | 0 | 0 |
| $\mu^-$ | 0 | 1 | 0 |
| $\mu^+$ | 0 | −1 | 0 |
| $\tau^-$ | 0 | 0 | 1 |
| $\tau^+$ | 0 | 0 | −1 |
| non − leptons | 0 | 0 | 0 |




$$e^+(\mathbf{p_2})e^-(\mathbf{p_1}) \rightarrow \mu^+(\mathbf{p_4})\mu^-(\mathbf{p_3})$$

- $L_e^i = 1 - 1 = 0 = L_e^f$
- $L_\mu^i = 0 = 1 - 1 = L_\mu^f$
- Initial state
- Final state
- Photon Propagator

## Transition Amplitude

$$\begin{aligned}
\frac{1}{i}T_{fi} &= \frac{1}{i}[\bar{v}(\mathbf{p_2})(-ie\gamma^\mu)u(\mathbf{p_1})\frac{-ig_{\mu\nu}}{k^2}\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)v(\mathbf{p_4})] \\
&= e^2[\bar{v}(\mathbf{p_2})\gamma^\mu u(\mathbf{p_1})\frac{g_{\mu\nu}}{s}\bar{u}(\mathbf{p_3})\gamma^\nu v(\mathbf{p_4})] \\
&= \frac{e^2}{s}[\bar{v}(\mathbf{p_2})\gamma^\mu u(\mathbf{p_1})\bar{u}(\mathbf{p_3})\gamma_\mu v(\mathbf{p_4})]
\end{aligned}$$

## useful formula

$$
\begin{aligned}
\gamma_0 &= g_{\mu 0}\gamma^0 &&= \gamma^0 \\
\gamma_k &= g_{\mu k}\gamma^k &&= -\gamma^k \\
\bar{u} &= u^\dagger \gamma^0 &&= u^\dagger \gamma_0 \\
(\gamma^\mu)^\dagger &= \gamma^0\gamma^\mu\gamma^0 && \\
(\gamma_\mu)^\dagger &= g_{\mu\nu}(\gamma^\nu)^\dagger = g_{\mu\nu}(\gamma^0\gamma^\nu\gamma^0) &&= \gamma^0\gamma_\mu\gamma^0 \\
\gamma^0\gamma^0 &= 1_4 &&
\end{aligned}
$$

## Element matrix squared

$$[\bar{v}(\mathbf{p_2})\gamma^\nu u(\mathbf{p_1})\bar{u}(\mathbf{p_3})\gamma_\nu v(\mathbf{p_4})]^\dagger$$

## Useful Formula

$$
\begin{array}{lcll}
\gamma_0 & = & g_{\mu 0}\gamma^0 & = \gamma^0 \\
\gamma_k & = & g_{\mu k}\gamma^k & = -\gamma^k \\
\bar{u} & = & u^\dagger \gamma^0 & = u^\dagger \gamma_0 \\
(\gamma^\mu)^\dagger & = & \gamma^0\gamma^\mu\gamma^0 & \\
(\gamma_\mu)^\dagger & = & g_{\mu\nu}(\gamma^\nu)^\dagger = g_{\mu\nu}(\gamma^0\gamma^\nu\gamma^0) & = \gamma^0\gamma_\mu\gamma^0 \\
\gamma^0\gamma^0 & = & 1_4 &
\end{array}
$$

## Insert

$$
\begin{aligned}
& [\bar{v}(\mathbf{p_2})\gamma^\nu u(\mathbf{p_1})\bar{u}(\mathbf{p_3})\gamma_\nu v(\mathbf{p_4})]^\dagger \\
= \; & [v^\dagger(\mathbf{p_2})\gamma^0\gamma^\nu u(\mathbf{p_1})u^\dagger(\mathbf{p_3})\gamma^0\gamma_\nu v(\mathbf{p_4})]^\dagger \\
= \; & [v^\dagger(\mathbf{p_4})(\gamma_\nu)^\dagger(\gamma^0)^\dagger(u^\dagger)^\dagger(\mathbf{p_3})u^\dagger(\mathbf{p_1})(\gamma^\nu)^\dagger(\gamma^0)^\dagger(v^\dagger)^\dagger(\mathbf{p_2})] \\
= \; & [v^\dagger(\mathbf{p_4})(\gamma_\nu)^\dagger(\gamma^0)^\dagger u(\mathbf{p_3})u^\dagger(\mathbf{p_1})(\gamma^\nu)^\dagger(\gamma^0)^\dagger v(\mathbf{p_2})] \\
= \; & [v^\dagger(\mathbf{p_4})\gamma^0\gamma_\nu\gamma^0\gamma^0\gamma^0\gamma^0 u(\mathbf{p_3})u^\dagger(\mathbf{p_1})\gamma^0\gamma^\nu\gamma^0\gamma^0\gamma^0\gamma^0 v(\mathbf{p_2})] \\
= \; & [v^\dagger(\mathbf{p_4})\gamma^0\gamma_\nu u(\mathbf{p_3})u^\dagger(\mathbf{p_1})\gamma^0\gamma^\nu v(\mathbf{p_2})] \\
= \; & [\bar{v}(\mathbf{p_4})\gamma_\nu u(\mathbf{p_3})\bar{u}(\mathbf{p_1})\gamma^\nu v(\mathbf{p_2})]
\end{aligned}
$$

## Formula

$$
\begin{aligned}
\sum_f M_{ff} &= Tr(M) \\
\sum_{spin} u\bar{u} &= \sum_{spin} v\bar{v} = \not{p} = p_\nu \gamma^\nu \\
Tr(\gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta) &= 4(g^{\alpha\beta} g^{\gamma\delta} + g^{\alpha\delta} g^{\beta\gamma} - g^{\alpha\gamma} g^{\beta\delta})
\end{aligned}
$$

## Matrix Element

$$
\begin{aligned}
|\mathcal{M}|^2 &= \frac{e^4}{4s^2} \sum [\bar{v}_a(\mathbf{p_2}) \gamma^\mu_{ab} u_b(\mathbf{p_1}) \bar{u}_c(\mathbf{p_3}) \gamma_{\mu cd} v_d(\mathbf{p_4})] \\
&\quad [\bar{v}_e(\mathbf{p_4}) \gamma_{\nu ef} u_f(\mathbf{p_3}) \bar{u}_g(\mathbf{p_1}) \gamma^\nu_{gh} v_h(\mathbf{p_2})] \\
&= \frac{e^4}{4s^2} \sum v_h(\mathbf{p_2}) \bar{v}_a(\mathbf{p_2}) \gamma^\mu_{ab} u_b(\mathbf{p_1}) \bar{u}_g(\mathbf{p_1}) \gamma^\nu_{gh} \\
&\quad u_f(\mathbf{p_3}) \bar{u}_c(\mathbf{p_3}) \gamma_{\mu cd} v_d(\mathbf{p_4}) \bar{v}_e(\mathbf{p_4}) \gamma_{\nu ef} \\
&= \frac{e^4}{4s^2} \sum_s Tr(v(\mathbf{p_2}) \bar{v}(\mathbf{p_2}) \gamma^\mu u(\mathbf{p_1}) \bar{u}(\mathbf{p_1}) \gamma^\nu) \\
&\quad Tr(u(\mathbf{p_3}) \bar{u}(\mathbf{p_3}) \gamma_\mu v(\mathbf{p_4}) \bar{v}(\mathbf{p_4}) \gamma_\nu) \\
&= \frac{e^4}{4s^2} Tr(\not{\mathbf{p}}_2 \gamma^\mu \not{\mathbf{p}}_1 \gamma^\nu) Tr(\not{\mathbf{p}}_3 \gamma_\mu \not{\mathbf{p}}_4 \gamma_\nu) \\
&= \frac{8e^4}{s^2} [(\mathbf{p_1 p_4})(\mathbf{p_2 p_3}) + (\mathbf{p_1 p_3})(\mathbf{p_2 p_4})]
\end{aligned}
$$

## Formula

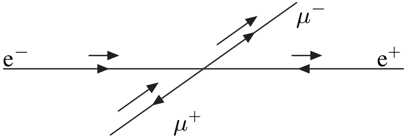
$$\begin{aligned}
(\mathbf{p_1}-\mathbf{p_3})^2 &= \mathbf{p_1}^2+\mathbf{p_3}^2-2\mathbf{p_1}\mathbf{p_3} \\
&= -2\mathbf{p_1}\mathbf{p_3} \\
&= -2\left(\frac{\sqrt{s}}{2}\frac{\sqrt{s}}{2}-\frac{\sqrt{s}}{2}\frac{\sqrt{s}}{2}\cos\theta\right) \\
(\mathbf{p_1}-\mathbf{p_4})^2 &= -2\frac{s}{4}(1+\cos\theta)
\end{aligned}$$

## Differential Cross section

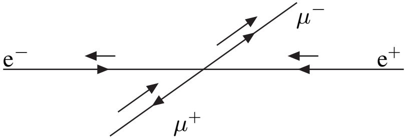
$$\begin{aligned}
\frac{d\sigma}{d\Omega} &= |\mathcal{M}|^2\frac{1}{64\pi^2 s} \\
&= \frac{8e^4}{64\pi^2 s^3}[(\mathbf{p_1}\mathbf{p_4})(\mathbf{p_2}\mathbf{p_3})+(\mathbf{p_1}\mathbf{p_3})(\mathbf{p_2}\mathbf{p_4})] \\
&= \frac{2\alpha^2}{s^3}[\frac{s}{4}(1+\cos\theta)\cdot\frac{s}{4}(1+\cos\theta) \\
&+ \frac{s}{4}(1-\cos\theta)\cdot\frac{s}{4}(1-\cos\theta)] \\
&= \frac{2\alpha^2}{s^3}[\frac{s^2}{16}(1+\cos\theta)^2+\frac{s^2}{16}(1-\cos\theta)^2] \\
&= \frac{\alpha^2}{4s}[1+\cos^2\theta]
\end{aligned}$$

$$\frac{d\sigma}{d\Omega} \sim (1 - \cos\theta)^2 + (1 + \cos\theta)^2$$

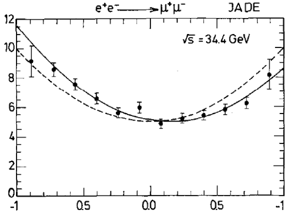
- Do the two terms have a particular meaning?
- Only the spin can lead to an angular distribution that is not flat
- Photon: Spin-1, mass zero $\rightarrow$ 2 dofs: $\pm 1$
- classical ED: 2 polarizations, no restframe...

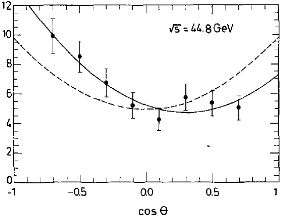


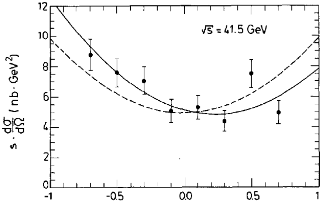

$$\begin{aligned} \theta(\mu^-, e^-) &= 0 \\ 1 + \cos\theta &= 2 \quad \text{Probmax} \end{aligned}$$




$$\begin{aligned} \theta(\mu^-, e^-) &= 0 \\ 1 - \cos\theta &= 0 \quad \text{Probmin} \end{aligned}$$

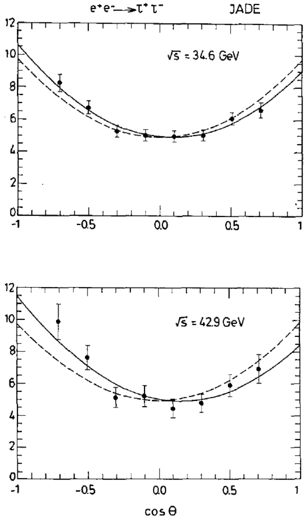


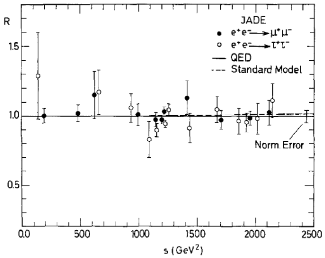




$e^+e^- \to \mu^+\mu^-$

- JADE detector at PETRA
- $s \cdot \frac{d\sigma}{d\Omega}$ scale invariant
- low $s \to (1 + \cos^2\theta)$
- higher $s \to$ asymmetry not QED




$e^+e^- \to \tau^+\tau^-$

- Small mass dependence at high $\sqrt{s}$
- Lepton universality
- Agreement with QED

Bohr

$$\begin{aligned}
\vec{\mu} &= Current \cdot Surface \cdot \vec{n} \\
&= \frac{e}{t} \cdot \pi r^2 \cdot \vec{n} \\
&= \frac{e}{2\pi r/v} \cdot \pi r^2 \cdot \vec{n} \\
&= \frac{e}{2m}(mvr)\vec{n} \\
&= \frac{e}{2m}(\hbar\ell)\vec{n} \\
&= \mu_B \ell \vec{n} \\
\mu_B &= 5.8 \cdot 10^{-5} eV/T
\end{aligned}$$

Intrinsic magnetic moment:

$$\vec{\mu} = g \cdot \mu_B \cdot \vec{S}$$

## Definition

$g$ is the gyromagnetic ratio, ratio of the magnetic dipole moment to the mechanical angular moment

Dirac

$$\begin{aligned}
\vec{J} &= \vec{L} + \vec{S} \\
&= \vec{L} + \frac{1}{2}\vec{\sigma} \\
\vec{\mu} &= \frac{1}{2}\int \vec{x} \times \vec{j} \\
\vec{j} &= -e\bar{\psi}\vec{\gamma}\psi \\
\langle f|\vec{\mu}|f\rangle &\sim \frac{1}{2}\langle f|\vec{j}|f\rangle \\
&= \frac{-e}{2}\langle f|\bar{\psi}\vec{\gamma}\psi|f\rangle \\
&= \frac{-e}{2}\langle f|\vec{L} + \vec{\sigma}|f\rangle \\
&= \frac{-e}{2}\langle f|\vec{L} + g\vec{S}|f\rangle
\end{aligned}$$

- The magnetic moment is anti-parallel with the Spin
- Dirac predicts $g = 2$!

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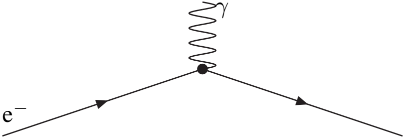
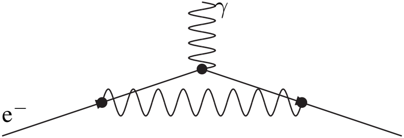
# and QFT?



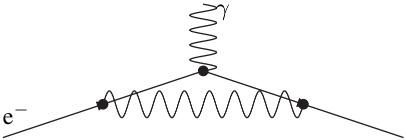

Interaction with an external field: LO




Interaction with an external field: NLO

Electromagnetic current

$$
\begin{aligned}
& -e\bar{u}\gamma^\mu u \\
= \quad & -\frac{e}{2m}\bar{u}[(p'+p)^\mu + i(p'-p)_\nu \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)]u \\
= \quad & -\frac{e}{2m}\bar{u}[(p'+p)^\mu + i(p'-p)_\nu \sigma^{\mu\nu}]u
\end{aligned}
$$




leads to:

$$\Delta\mu \sim \alpha/\pi \cdot \frac{e}{2m}$$
$$g = 2 + \alpha/\pi$$
$$a = \frac{g-2}{2} = \frac{1}{2}\frac{\alpha}{\pi} \sim 10^{-3}$$

| *Order* | *Diagrams* |
|---|---|
| 1 | 1 |
| 2 | 7 |
| 3 | 72 |
| 4 | 891 |
| 5 | 12672 |

QED prediction $a_e$

$$a_e = 1159652182.79 \cdot 10^{-12} \pm 7.79 \cdot 10^{-12}$$

8th order: Phys. Rev. Lett. 99, 110406 (2007)

## Electron Precession in B-field

$$
\begin{aligned}
mv_p^2/r &= ev_pB \\
mv_p/r &= eB \\
m\omega r/r &= eB \\
\omega_0 &= eB/m \\
m &\rightarrow m\gamma \\
\omega_C &= \omega_0/\gamma
\end{aligned}
$$

## Spin Precession in B-field

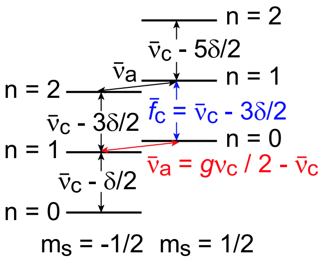
Magnetic torque:
$\Delta E = g\mu_B B = \hbar\omega_L$
$\omega_L = g(eB)/(2m) = \frac{1}{2}g\omega_0$
Relativistic corrections (Thomas):
$\omega_P = \omega_L - \omega_T = \frac{g}{2}\omega_0 - \frac{\gamma-1}{\gamma}\omega_0$

## Phase difference

$\Delta\omega = \omega_L - \omega_0 = a_e\omega_0$
Relativistic: $\Delta\omega = \omega_P - \omega_C = a_e\omega_0$

## $a_e$

$a_e = 0$ : Spin in phase with electron rotation
$a_e \neq 0$ : Spin precession not in phase with precession of particle in B-field

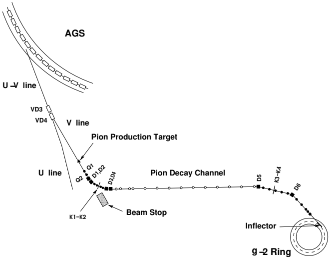


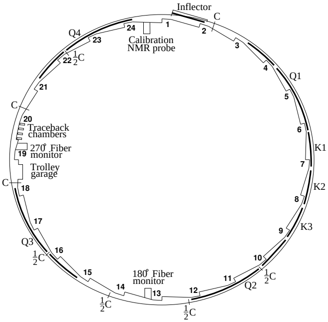

- Penning trap electrons (small scale experiment)
- $\delta/\nu_C$: relativistic shift
- f Cyclotron : 149 GHz
- f Anomaly : 173 MHZ

$a_e$

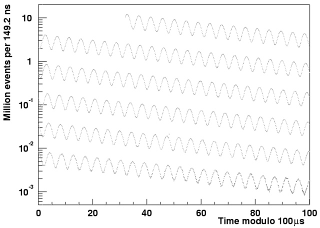
$$a_e = 115965218073(28) \cdot 10^{-14}$$
$$\alpha^{-1} = 137.035999084(51)$$

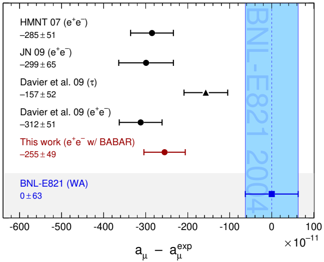
- test QED to $10^{-13}$
- determine $\alpha$ to 0.37ppb ($\approx 10^{-9}$)
- natural scale: $m_e \approx 0.5 MeV$




- muon liftime penning trap not feasible
- 24GeV protons to produce pions (next week) which decay to muons
- muons decay to electrons
- calorimeters detect the electrons
- excellent knowledge of B-field necessary




- electron counting rate varies as function of the precession of the spin
- natural scale of experiment $m_\mu \approx 0.105 GeV$

- Hadronic contribution (non QED) important (695)
- Prediction is mixture of calculation and measurement
- Supersymmetry?