Particle Physics
The Standard Model
Overview of the Standard Model

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Part I

Overview of the Standard Model

Metric

A four-vector **x** is attributed to a particular space-time point.

$$\mathbf{x} = (x^\mu) = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} t \\ \vec{x} \end{pmatrix}$$

Greek letters are for four-vectors
Roman letters for spatial coordinates (vectors)
The scalar product is defined thanks to the metric tensor $g^{\mu\nu}$

$$\mathbf{g} = (g_{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

by

$$\mathbf{x}.\mathbf{y} = g_{\mu\nu} x^\mu y^\nu = x^\mu y_\mu = x_\mu y^\mu$$

Lorentz transformation

- A transformation (Λ, a) defines the transition from an inertia frame to another
$$(\Lambda, a) : x^\mu \to x'^\mu = \Lambda^\mu_\nu x^\nu + a^\mu$$
- The energy and 3-momentum p of a particle of mass m form a four-vector whose square $\mathbf{p}.\mathbf{p} = m^2$
- In the course, we will apply the Einstein summation rule on greek indices
- The velocity of the particle is $\beta = v/c = p/E$
- and the Lorentz factor is $\gamma = \frac{1}{\sqrt{1-\beta^2}}$
- The energy and momentum $(E^\star, p^\star)$ viewed from a frame moving with velocity β_f are given by

$$\begin{pmatrix} E^\star \\ p^\star_{||} \end{pmatrix} = \begin{pmatrix} \gamma_f & -\gamma_f\beta_f \\ -\gamma_f\beta_f & \gamma_f \end{pmatrix} \begin{pmatrix} E \\ p_{||} \end{pmatrix}$$

Special relativity - Space-time coordinates

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

with

$$\beta = v/c \quad \text{and} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Matter = fermions (Spin-$\frac{1}{2}$ particles):

- Electrons with two spin orientations: L and R
- Neutrinos (L)
- Quarks L and R (proton=uud, neutron=udd)
- Three families = heavier copies of the first family

$$\begin{pmatrix} \mathrm{u_L} \\ \mathrm{d_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{c_L} \\ \mathrm{s_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{t_L} \\ \mathrm{b_L} \end{pmatrix}$$

$$\begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_\mathrm{L}} \\ \mu_\mathrm{L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_\mathrm{L}} \\ \tau_\mathrm{L} \end{pmatrix}$$

$$\begin{matrix} \mathrm{u_R} & \mathrm{c_R} & \mathrm{t_R} \\ \mathrm{d_R} & \mathrm{s_R} & \mathrm{b_R} \\ \mathrm{e_R} & \mu_\mathrm{R} & \tau_\mathrm{R} \end{matrix}$$

Interactions = bosons (Spin–0 or –1 particles):

- Electromagnetism: Spin–1 massless
- Strong interaction (p=uud): Spin–1 massless
- Weak interaction: Spin–1 massive
- Masses: Spin–0 massive

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$$

$$\begin{matrix} u_R & c_R & t_R \\ d_R & s_R & b_R \\ e_R & \mu_R & \tau_R \end{matrix}$$

$$\begin{matrix} \gamma \\ g \\ W^{\pm}, Z^{\circ} \\ H \end{matrix}$$

- Fractional charges not observed in nature
- Strong interaction: uud, udd

$$\begin{array}{cccc}
\begin{matrix} \frac{2}{3} \\ -\frac{1}{3} \end{matrix} & \begin{pmatrix} u_L \\ d_L \end{pmatrix} & \begin{pmatrix} c_L \\ s_L \end{pmatrix} & \begin{pmatrix} t_L \\ b_L \end{pmatrix} \\
\begin{matrix} 0 \\ -1 \end{matrix} & \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} & \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} & \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix} \\
\frac{2}{3} & u_R & c_R & t_R \\
-\frac{1}{3} & d_R & s_R & b_R \\
-1 & e_R & \mu_R & \tau_R \\
0 & \gamma & & \\
0 & g & & \\
\pm 1, 0 & W^{\pm}, Z^{\circ} & & \\
0 & H & &
\end{array}$$

- Sum of colours (RGB) white
- R+G+B= (qqq =baryon)
- Colour+anti-colour= White ($q\bar{q}$ =meson)
- Gluon carries colour+anti-colour
- 8 different gluons (not 9)

$$\begin{array}{cccc}
C & \begin{pmatrix} u_L \\ d_L \end{pmatrix} & \begin{pmatrix} c_L \\ s_L \end{pmatrix} & \begin{pmatrix} t_L \\ b_L \end{pmatrix} \\
C & & & \\
- & \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} & \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} & \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix} \\
- & & & \\
C & u_R & c_R & t_R \\
C & d_R & s_R & b_R \\
- & e_R & \mu_R & \tau_R
\end{array}$$

$$\begin{array}{cc}
- & \gamma \\
C + \bar{C}' & g \\
- & W^{\pm}, Z^{\circ} \\
- & H
\end{array}$$

Rule of thumb for interactions

Interaction	Carrier	Relative strength
Gravitation	Graviton (G)	10^{-40}
Weak	Weak Bosons ($W^{\pm}$,Z°)	10^{-7}
Electromagnetic	Photon (γ)	10^{-2}
Strong	Gluon (g)	1

- Forget about Gravitation in particle physics problems
- The course will lead us to understand how the model describes the interactions and their strength.

$\mathbf{a} = (E_a, \vec{p}_a) = (p_0, p_1, p_2, p_3)$
$E_a \cdot E_a - \vec{p}_a \cdot \vec{p}_a = m_a^2$
$\mathrm{g}^{\mu\nu} p_\mu p_\nu = m_a^2$

$\mathrm{g}^{\mu\mu} = (1, -1, -1, -1)$
for $\mu \neq \nu : \mathrm{g}^{\mu\nu} = 0$

Conservation of E and $\vec{p}$

$$\mathbf{a} + \mathbf{b} = \mathbf{c} + \mathbf{d}$$

therefore

$$\mathbf{a} - \mathbf{c} = \mathbf{d} - \mathbf{b}$$

Mandelstam Variables

$$a + b \rightarrow c + d$$

$$s = (\mathbf{a} + \mathbf{b})^2$$

$$t = (\mathbf{a} - \mathbf{c})^2$$

$$u = (\mathbf{a} - \mathbf{d})^2$$

Theorem

$$s+t+u$$
$$= \quad m_a^2+m_b^2+m_c^2+m_d^2$$
$$= \quad 0$$

- High energy approx ($E \gg m \sim 0$, $E = |\vec{p}|$)
- CM-frame ($\vec{p}_a = -\vec{p}_b$)
- $\rightarrow E_a = E_b = E_c = E_d = \sqrt{s}/2$

Proof.

$$\begin{aligned}
s &= \mathbf{a}^2+\mathbf{b}^2+2\cdot\mathbf{a}\cdot\mathbf{b} \\
&= m_a^2+m_b^2+2(E_a\cdot E_b-\vec{p}_a\cdot\vec{p}_b) \\
&= 2(E_a\cdot E_b-\vec{p}_a\cdot\vec{p}_b) \\
&= 2(E_a\cdot E_a+\vec{p}_a\cdot\vec{p}_a) \\
&= 2(E_a^2+E_a^2) \\
&= 4E_a^2 \\
t &= -2(E_a\cdot E_c-\vec{p}_a\cdot\vec{p}_c) \\
u &= -2(E_a\cdot E_d-\vec{p}_a\cdot\vec{p}_d) \\
&= -2(E_a\cdot E_c+\vec{p}_a\cdot\vec{p}_c)
\end{aligned}$$

□

Proof.

$$\begin{aligned} t + u &= -2(2 \cdot E_a \cdot E_c) \\ &= -2(2 \cdot E_a \cdot E_a) \\ s + t + u &= 4 \cdot E_a \cdot E_a - 4 \cdot E_a \cdot E_a \\ &= 0 \end{aligned}$$

□

2 particle reaction → 2 independent variables!
The proof is easy to get without the high energy approximation (exercise)

Useful relationships

$$
\begin{aligned}
t &= -2(E_a \cdot E_c - \vec{p}_a \cdot \vec{p}_c) \\
&= -2(\tfrac{\sqrt{s}}{2} \cdot \tfrac{\sqrt{s}}{2} - \tfrac{\sqrt{s}}{2} \cdot \tfrac{\sqrt{s}}{2} \cdot \cos\theta) \\
&= -\tfrac{s}{2} \cdot (1 - \cos\theta) \\
u &= -\tfrac{s}{2} \cdot (1 + \cos\theta) \\
t &= -2(\tfrac{\sqrt{s}}{2} \cdot \tfrac{\sqrt{s}}{2} - \tfrac{\sqrt{s}}{2} \cdot \sqrt{E_c^2 - m_c^2} \cdot \cos\theta) \\
&= -2(\tfrac{\sqrt{s}}{2} \cdot \tfrac{\sqrt{s}}{2} - \tfrac{\sqrt{s}}{2} \cdot \sqrt{s/4 - m_c^2} \cdot \cos\theta) \\
&= -2(\tfrac{\sqrt{s}}{2} \cdot \tfrac{\sqrt{s}}{2} - \tfrac{\sqrt{s}}{2} \cdot \tfrac{\sqrt{s}}{2} \cdot \sqrt{1 - 4m_c^2/s} \cdot \cos\theta) \\
&= -\tfrac{s}{2} \cdot (1 - \beta \cdot \cos\theta)
\end{aligned}
$$

massless massless initial state and massive final state of identical particles

Crossing relationship

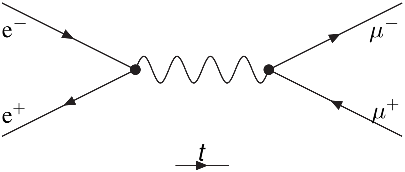
$$
\begin{array}{l|llll}
a+b\to c+d & a+\bar{c}\to\bar{b}+d & & \\
s=(\mathbf{a}+\mathbf{b})^2 & s'=(\mathbf{a}+\bar{\mathbf{c}})^2 & =(\mathbf{a}-\mathbf{c})^2 & =t \\
t=(\mathbf{a}-\mathbf{c})^2 & t'=(\mathbf{a}-\bar{\mathbf{b}})^2 & =(\mathbf{a}+\mathbf{b})^2 & =s \\
u=(\mathbf{a}-\mathbf{d})^2 & u'=(\mathbf{a}-\mathbf{d})^2 & =(\mathbf{a}-\mathbf{d})^2 & =u
\end{array}
$$

- Calculate a process as function of s,t,u
- Derive crossed process by $s\to t$, $t\to s$, $u\to u$
- We can express one process in the kinematic variables of another process (Xcheck)
- Global factor -1 for each fermion line crossed (will see an example)

Kinematics in Tutorial

s–channel: annihiliation

$$e^+ + e^- \to \gamma \to \mu^+\mu^-$$

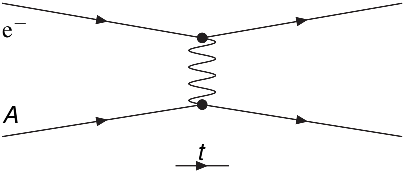


$$\begin{aligned} \mathbf{q}_\gamma &= \mathbf{p}_{e^-} + \mathbf{p}_{e^+} \\ s &= \mathbf{q}_\gamma^2 \\ (CM) &= (E_{e^-} + E_{e^+})^2 \\ &> 0 \end{aligned}$$

the photon is massive (virtual)
time-like

t–channel: scattering

$$e^- + A \to e^- + A$$



$$\begin{aligned} \mathbf{p}_{e_i^-} &= \mathbf{q}_\gamma + \mathbf{p}_{e_o^-} \\ t &= \mathbf{q}_\gamma^2 \\ &= m_e^2 + m_e^2 - 2 \cdot \mathbf{p}_{e_i^-} \cdot \mathbf{p}_{e_o^-} \\ &\approx -2(E_i E_o - |\vec{p}_i||\vec{p}_o| \cos\theta) \\ &\approx -2E_i E_o(1 - \cos\theta) \\ &\leq 0 \end{aligned}$$

the photon is massive space-like

Cross Section

- The cross section σ is the ratio of the transition rate and the flux of incoming particles.
- Its unit is cm^2
- $1\mathrm{b} = 10^{-24}\mathrm{cm}^2$ (puts barn in perspective, doesn't it?)

Two ingredients:

- the interaction transforming initial state $|i\rangle$ to a final state $\langle f|$ of m particles with four-vectors $\mathbf{p'_i}$
- kinematics (including Lorentz-Invariant phase space element)

$$\mathrm{d}\sigma = \frac{1}{2S_{12}} \prod_{i=1}^{m} \frac{\mathrm{d}^3 p'_i}{(2\pi)^3 2E'^0_i} (2\pi)^4 \delta(\mathbf{p'_1} + \ldots + \mathbf{p'_m} - \mathbf{p_1} - \mathbf{p_2}) |\mathcal{M}|^2$$

with (originating from flux) $S_{12} = \sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}$

Total Width or Decay Rate

- Total width is the inverse of the lifetime of the particle
- unit: energy, e.g., GeV.
- Closely related, but not identical to the cross section

$$\mathrm{d}\Gamma = \frac{1}{2E}\prod_{i=1}^{m}\frac{\mathrm{d}^3 p_i'}{(2\pi)^3 2E_i'^0}(2\pi)^4\delta(\mathbf{p_1'} + ... + \mathbf{p_m'} - \mathbf{p_1})|\mathcal{M}|^2$$

For the decay of an unpolarized particle of mass M into two particles (in the CM frame $\vec{p}_1' = -\vec{p}_2'$):

$$\mathrm{d}\Gamma = \frac{1}{32\pi^2}\frac{|\vec{p}_1'|}{M^2}|\mathcal{M}|^2\mathrm{d}\Omega$$

where Ω is the solid angle with $\mathrm{d}\Omega = \mathrm{d}\phi\mathrm{d}\cos\theta$

Cross section and total width for a final state with 2 particles

Cross section $2 \to 2$ reaction with four massless particles:

$$d\sigma = \frac{1}{64\pi^2}\frac{|\mathcal{M}|^2}{s}d\Omega$$

Width of a massive particle ($\sqrt{s} = M$) decaying to two massless particles in the final state $|\vec{p}_1'| = \sqrt{s}/2$:

$$d\Gamma = \frac{1}{64\pi^2}\frac{|\mathcal{M}|^2}{\sqrt{s}}d\Omega$$

Study of the phase space in Problem Solving with applications to 2-body.

- Particles: plane waves
  $\psi(\vec{x}, t) \sim \exp -im_0 t$
- $m_0 \rightarrow m_0 - i\Gamma/2$

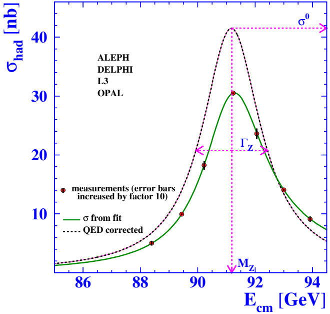
$$\begin{aligned} N(t) &= N_0 \cdot \exp -t/\tau \\ \Gamma &= 1/\tau \end{aligned}$$

Fourrier transform to momentum space:

$$\begin{aligned} A &\sim \frac{1}{(m-m_0)+i\Gamma/2} \\ |A|^2 &\sim \frac{1}{(m-m_0)^2+\Gamma^2/4} \end{aligned}$$

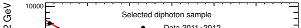
Γ: full width half maximum
Similarity to classical mechanics: resonance

Example $e^-e^+ \rightarrow Z^\circ \rightarrow q\bar{q}$



lifetime too short to be measured directly: measure mass via decay products $q\bar{q}$
cross section measurement

Example $pp \rightarrow H \rightarrow \gamma\gamma$ (EW 2013)



Suppose that we have two (and exactly two) possible decays for the particle a:

$$\begin{aligned} a &\rightarrow b + c \\ a &\rightarrow d + e \end{aligned}$$

then:

$$\Gamma = \Gamma_{bc} + \Gamma_{de}$$

If a particle of a given mass can decay to more final states than another one with the same mass, it will have a shorter lifetime

Branching ratio

$\mathcal{B}(\mathrm{a} \rightarrow \mathrm{b} + \mathrm{c}) = \Gamma_{bc}/\Gamma$
The branching ratio: Of N decays of particle a, a fraction $\mathcal{B}$ will be the final state with the particles b and c. Γ_{bc} is a partial width of particle a.

Remember: for the calculation Γ ALL final states (partial widths) have to be considered.

What do we know?

- Names of particles
- Kinematic description of interactions
- Definition of cross section and decay width

What is next?

- Electromagnetic interactions (QED)
- Strong interaction (QCD)
- Electroweak interactions