The Number of Neutrinos and the Z Line Shape

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Abstract

Thanks to a precise calibration and monitoring of the LEP beam energy the Z resonance line shape has been precisely measured, hence permitting accurate determinations of the Z mass and width. A large effort was spent by the experiments for the measurement of the LEP luminosity, enabling the number of neutrino types to be precisely determined. The result, $N_\nu = 2.9841 \pm 0.0083$, demonstrates the existence of only 3 neutrino species.

La Résonance Z et le nombre de neutrinos

Grâce à la calibration précise de l'énergie de LEP et de son suivi dans le temps, il a été possible de reconstruire la courbe de résonance du boson Z et d'en tirer des valeurs très précises pour sa masse et sa largeur. Les expériences sur LEP ont beaucoup investi dans la qualité de la mesure de la luminosité de la machine, ce qui a permis la mesure précise du nombre de types de neutrinos. Le résultat, $N_\nu = 2.9841 \pm 0.0083$, prouve l'existence de seulement 3 neutrinos.

1 Introduction: what is the number of families of fermions?

At the time LEP started, the basic properties of the weak interactions were already well known. One pressing question, however, could not be answered either by theoretical arguments or by direct experiments: what is the number of families of fermions? LEP answered this fundamental question in a few weeks by measuring the Z resonance. With six years of data and meticulous measurements of luminosity and energy the LEP experimentalists determined the Z boson mass and width, as well as the Z decay rates, with a precision which is unlikely to be surpassed soon.

All elementary quarks and leptons that have been observed are organised in *exactly* three families: (or generations).

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix} \qquad \mathrm{doublets\ of\ left-handed\ quarks}$$

$$\begin{matrix} (u) \\ (d) \end{matrix} \qquad \begin{matrix} (c) \\ (s) \end{matrix} \qquad \begin{matrix} (t) \\ (b) \end{matrix} \qquad \mathrm{singlets\ of\ right-handed\ quarks}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \qquad \mathrm{doublets\ of\ left-handed\ leptons}$$

$$\begin{matrix} (?\nu_e) \\ (e) \end{matrix} \qquad \begin{matrix} (?\nu_\mu) \\ (\mu) \end{matrix} \qquad \begin{matrix} (?\nu_\tau) \\ (\tau) \end{matrix} \qquad \mathrm{singlets\ of\ right-handed\ leptons}$$

As far as we can tell the electroweak theory could not easily accomodate further isolated fermions, but it could accomodate any number of families of the same type. One could easily envisage a situation where many families including heavy charged quarks and leptons could exist, without these heavy leptons being ever produced in accessible experiments, by lack of available energy. Nevertheless, since the

known neutrinos are very light, it is natural to expect that these additional families would include light neutrinos as well, leading to the possibility that many families of light neutrinos would exist.

The existence of many light neutrinos would have considerable cosmological consequences. In particular, the evolution of the universe within the first second after the Big Bang would be profoundly affected. The argument, developed in [1], is the following. At the time where energies are large enough, reactions such as $e^+e^- \to \nu\bar\nu$ transform a fraction of available energy into neutrinos in a democratic way. The creation of neutrons and protons however is controled by reactions involving the electron neutrino, such as $\nu_e + n \to p + e^-$, and is consequently very sensitive to the number of light neutrino families, N_ν, which compete with electron neutrinos. The relative abundance of Hydrogen, Deuterium and Helium, and therefore the entire chemical constitution of our universe is a sensitive function of this number.

Before SLC and LEP started, limits on the number of light neutrinos were given from the above cosmological considerations, since there are data on the relative abundance of various nuclei in the universe, in particular the ratio of helium to hydrogen, or, with similar arguments, from the time development of the supernova 1987A. There were also indications from the direct search for the process $e^+e^- \to \nu\bar\nu\gamma$ (single-photon experiments), or from the early measurements of the Z and W boson properties in the CERN and FERMILAB $p\bar p$ experiments. A review of these constraints published in 1989 [2] evaluated the best estimate of N_ν to be $N_\nu = 2.1^{+0.6}_{-0.4}$, and stated that "$N_\nu = 3$ is perfectly compatible with all data, but four families still provide a reasonable fit".

In searching for further families of neutrinos, it will be assumed that their couplings are the same as those of ν_e, ν_μ and ν_τ. Universality is deeply embedded in the Standard Model, identical multiplets having the same coupling constant. It is very well verified for Charged Current interactions of e-μ-τ leptons, including the neutrinos, and for Neutral Current interactions of charged leptons (see the contribution by A. Rougé *et al.*).

2   Determination of the number of light neutrino species a LEP and SLC

The most precise determination of the number of light neutrino species is obtained from measurements of the visible cross sections of e^+e^- annihilation at and around the Z resonance, as is made explicit in Figure 1. If the Z is allowed to decay into more types of light neutrinos which will lead to an unvisible final state, it will decay less often into the visible ones.

The realization that the visible cross sections might be sensitive to the number of light neutrinos is rather ancient, one finds the question asked in John Ellis "Zedology" [3], where one can find this anguished question: *The Z peak is large and dramatic, as long as there are not too many generations of fermions. Is it conceivable that there might be so many generations as to wash out the Z peak?* Since at that time the bound on the number of light neutrinos was very weak (about 6000), this certainly was a frightening possibility for those planning to build LEP! Dramatic also were the few first weeks of SLC and LEP operation where it was quickly realized that the Z peak was there indeed, large and dramatic, and that, alas, the number of light neutrinos was three.

There was intense competition between the SLC at SLAC (California, USA) and LEP at CERN (Switzerland). The two projects were rather different in concept. LEP was build as the largest possible conventional e^+e^- storage ring, with a circumference of 27 km. This standard technique would ensure few surprises and reliable high luminosity. SLC on the other hand, was the prototype of a new concept of accelerator, the linear collider; it was re-using the old Stanford linac, with improvements in the acceleration technique (RF pulse compression) and addition of arcs to bring e^+ and e^- in collisions, as well as of challenging positron source and damping rings.

The commissioning of SLC started in early 1987, and lead to a number of technical difficulties, not surprising in retrospect for such a new project. The first Z hadronic decay was produced on April 11 1989, and recorded in the MarkII detector. Luminosity was very low, a few 10^{-27}/cm^2/s, leading to a few Z hadronic decays per day. With the LEP start-up advertised for the 14 of July, the time where SLC
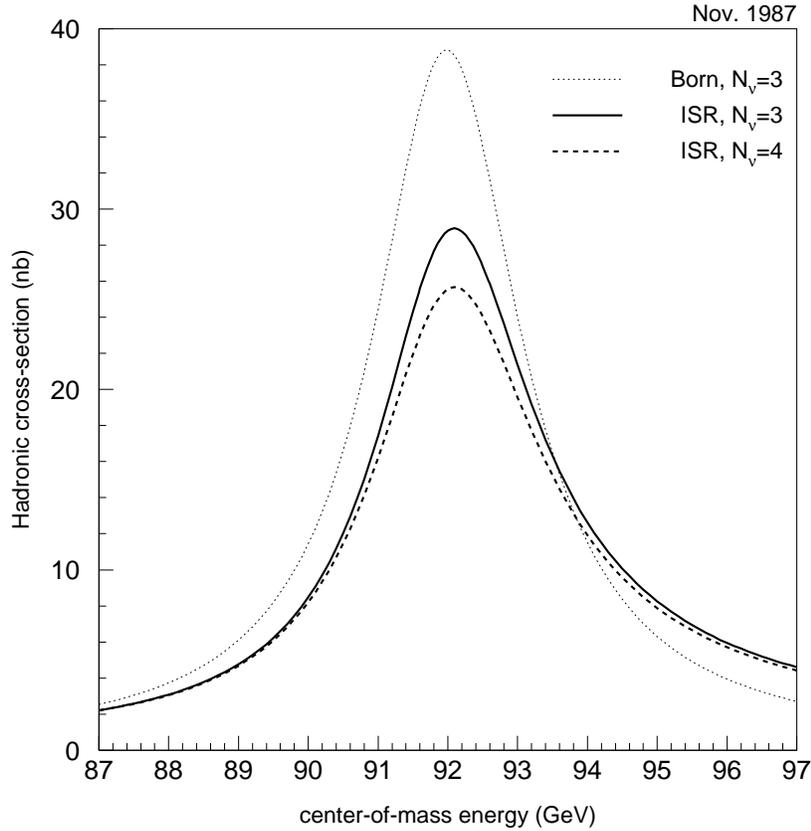
Fig. 1: *The $e^+ e^- \to hadrons$ cross section as a function of center-of-mass energy. This curve was drawn before LEP start-up in 1987. At that time the Z mass was measured to be around 92 GeV/c^2 with an error larger than 1.5 GeV/c^2. The doted line represents the Born approximation prediction for three species of light neutrinos. The full line includes the effect of initial state radiation. The dashed line represents the effect of adding one more type of light neutrino with the same couplings as the first three. It is clear from this picture that the cross section at the peak of the resonance contains most of the information on the number of light neutrino species.*

would hold the lead was going to be short, and intense. Nevertheless the SLC collaboration was able to collect a total of 106 Z decays by 24 July and submit a publication [4], where the Z mass was determined to be $M_Z = 91.11 \pm 0.23$ GeV/c^2, and the number of light neutrinos species $N_\nu = 3.8 \pm 1.4$.

LEP did not start collisions on 14 of July but on 13 August for one week. The high luminosity optics were however not yet commissioned and events came at a rate of 1 a day for the four experiments; this was not enough to make a measurement. Running resumed on 20 September with superconducting quadrupoles and in just three weeks, until 9 October, 3000 Z's were collected in each of the experiments. By 13 October, a seminar was organised at CERN where the four collaborations presented their first results [5, 6, 7, 8, 9], shown in Table 1. The day before, SLC had organised a public conference where updated results had been presented [9], based on 480 events. The "online average" of these results is also shown in Table 1, $N_\nu = 3.12 \pm 0.19$. The number of light neutrinos was three.

Following this important contribution, SLC was shaken by an earthquake on 24 October 1989, from which it took more than a year to recover, and then concentrated on polarised beam physics. LEP went on, to the end of 1989 and for 6 more years (1989 to 1995), each experiment collecting 4 million hadronic Z decays. With the final results now available, the number of light neutrinos was determined to

Experiment	hadronic Zs	Z mass (GeV/c^2)		N_ν	
MARKII	450	91.14 $\pm$ 0.12	2.8	$\pm$	0.60
L3	2538	91.13 $\pm$ 0.06	3.42	$\pm$	0.48
ALEPH	3112	91.17 $\pm$ 0.05	3.27	$\pm$	0.30
OPAL	4350	91.01 $\pm$ 0.05	3.10	$\pm$	0.40
DELPHI	1066	91.06 $\pm$ 0.05	2.4	$\pm$	0.64
Average		91.10 $\pm$ 0.05	3.12	$\pm$	0.19

Table 1: *First results from LEP and SLC on the Z mass and the number of light neutrino species, as published around 12 October 1989 (in order of submission to the journal).*

be [10]

$$N_\nu = 2.9841 \pm 0.0083 . \tag{1}$$

The early results were unexpectedly precise, and the precision of final ones exceed by a factor 100 the expectations that could be found in the studies preceeding the start of LEP. Once the method is explained in more detail, it will become clear that the unexpected capacity of the experiments to perform precise measurement of hadronic cross sections is the reason for this success.

3 Determination of the Z line shape parameters

Around the Z pole, the $e^+e^- \rightarrow Z \rightarrow f\bar{f}$ annihilation cross section is given by

$$\sigma_f = \frac{12\pi(\hbar c)^2}{M_Z^2} \frac{s\Gamma_e\Gamma_f}{(s - M_Z^2)^2 + s^2 \frac{\Gamma_Z^2}{M_Z^2}}. \tag{2}$$

which is a general formula for the production of a spin one particle in e^+e^- annihilation, decaying into a visible channel f. This typical resonance shape peaks around the Z mass, $\sqrt{s} = m_Z$, and has a width Γ_Z. If the Z decays a fraction B_f of the time into a final state f the corresponding partial width is defined as $\Gamma_f = B_f\Gamma_Z$.

The Standard Model predicts the numerical values for the Z partial widths, as displayed in table 2. The main decay mode of the Z is into hadrons (70%), each of the leptons representing only 3% and three neutrinos would contribute 20%. There are no other substantial decay modes unless there are new particles; the Higgs branching ratio, in particular, is very small. In this expression the number of neutrinos intervenes through the total Z width Γ_Z:

$$\Gamma_Z = 3\Gamma_\ell + \Gamma_{\text{had}} + N_\nu\Gamma_\nu \tag{3}$$

If N_ν increases, the total width which is in the denominator of equation 2 increases, and the cross section is decreased.

Equation 2 receives a number of modifications to account for the contribution of the photon exchange process (this is less than one percent), and more importantly what is called 'initial state radiation' (ISR), in which one or both of the initial state electrons looses energy into photons. This phenomenon reduces the initial state energy and smears out the resonance significantly as can be seen in Figure 1. Due to the availability of calculations up to second order in pertrbation theory, this large (30% at the peak) effect can be corrected for with a relative precision of $5 \ 10^{-4}$.

The principle of the analysis is then as follows: all visible channels are detected by large acceptance detectors and classified according to four categories: i) hadrons, ii) electron pairs, iii) muon pairs,

f	I_{3f}	Q_f	g_{Af}	g_{Vf}	Γ_f (MeV/c^2)
ν	1/2	0	1/2	1/2	167
e	-1/2	-1	-1/2	-0.04	84
u	1/2	2/3	1/2	0.19	300
d	-1/2	-1/3	-1/2	-0.35	383
b	-1/2	-1/3	-1/2	-0.35	376
hadrons $= u + d + c + s + b$	–	–	–	–	1740
total for 3 neutrinos	–	–	–	–	2500
total for 4 neutrinos	–	–	–	–	2670

Table 2: *Numerical values of quantum numbers, neutral current couplings, and Z decay partial widths, for the four types of fermions, for hadrons and total width. The value of* $\sin^2 \theta_{\mathrm{w}}^{\mathrm{eff}}$ *is 0.2315.*

iv) tau pairs. Examples of such events are shown in Figure 2. These events are easy to detect, with high and well known efficiencies (as high as $(99 \pm 0.05)\%$ for hadronic decays), and to separate from each other.

In order to extract a cross section from the number of events, the luminosity of the accelerator needs to be determined ($N = \mathcal{L}.\sigma$). This is done by measuring at the same time another process with a calculable cross section, the elastic scattering $e^+ e^- \to e^+ e^-$, known as Bhabha scattering, which results in two low angle electron and positron. To this effect, the LEP experiments were equipped ab initio with low angle detectors, to detect these scattered electrons. The precision with which these detectors can measure this process is determined by the knowledge of the solid angle they cover and by the accuracy with which they measure the angle of the scattered electrons. The initial detectors were able to reach a precision of about one percent, but were progressively replaced with extremely precisely machined silicon tungsten calorimeters or silicon trackers (as in Figure 3 which allowed a determination of the luminosity with an accuracy of $5 \ 10^{-4}$ or better. It took many years of detailed higher order calculations to achieve a similar precision on the theoretical estimate of the cross section within this well defined acceptance.

Measurements of cross section for a given final state $f\bar{f}$ around the Z pole allows to extract three parameters: the position of the peak, the width of the resonance and an overall normalisation, that is best obtained from the peak cross section,

$$\sigma_f^0 = \frac{12\pi (\hbar c)^2}{\mathrm{M}_\mathrm{Z}^2} \frac{\Gamma_e \Gamma_f}{\Gamma_\mathrm{Z}^2} = \frac{12\pi (\hbar c)^2}{\mathrm{M}_\mathrm{Z}^2} B_e \, B_f \ . \tag{4}$$

By measuring cross sections for hadrons, electron pairs, muon pairs and tau pairs, one can obtain six numbers: the mass, the total width, and four other parameters which could be four branching ratios, or four partial widths. A better choice is to use the peak cross section for hadrons, corrected for initial state photon radiation, $\sigma_{\mathrm{had}}^{\mathrm{peak},0}$, and the ratios of hadrons to the various leptonic partial widths, $\mathrm{R}_\ell \equiv \Gamma_{\mathrm{had}}/\Gamma_\ell$. The Standard Model implies lepton universality, and if this is assumed, the number of parameters can be reduced to four, $m_\mathrm{Z}, \Gamma_\mathrm{Z}, \sigma_{\mathrm{had}}^{\mathrm{peak},0}, \mathrm{R}_\ell$. The choice of these observables to fit the line-shape measurements is dictated by the fact that they are experimentally uncorrelated, both from the point of view of statistical and systematic errors.

By reporting the expression for Γ_Z of Eq. 3 into the peak cross section for hadrons, eq. 4, the number of neutrinos can be extracted from quantities that are measured at the peak only:

$$\mathrm{N}_\nu = \frac{\Gamma_\ell}{\Gamma_\nu} \left(\sqrt{\frac{12\pi \mathrm{R}_\ell}{\mathrm{M}_\mathrm{Z}^2 \sigma_{\mathrm{had}}^{\mathrm{peak},0}}} - \mathrm{R}_\ell - 3 \right) . \tag{5}$$
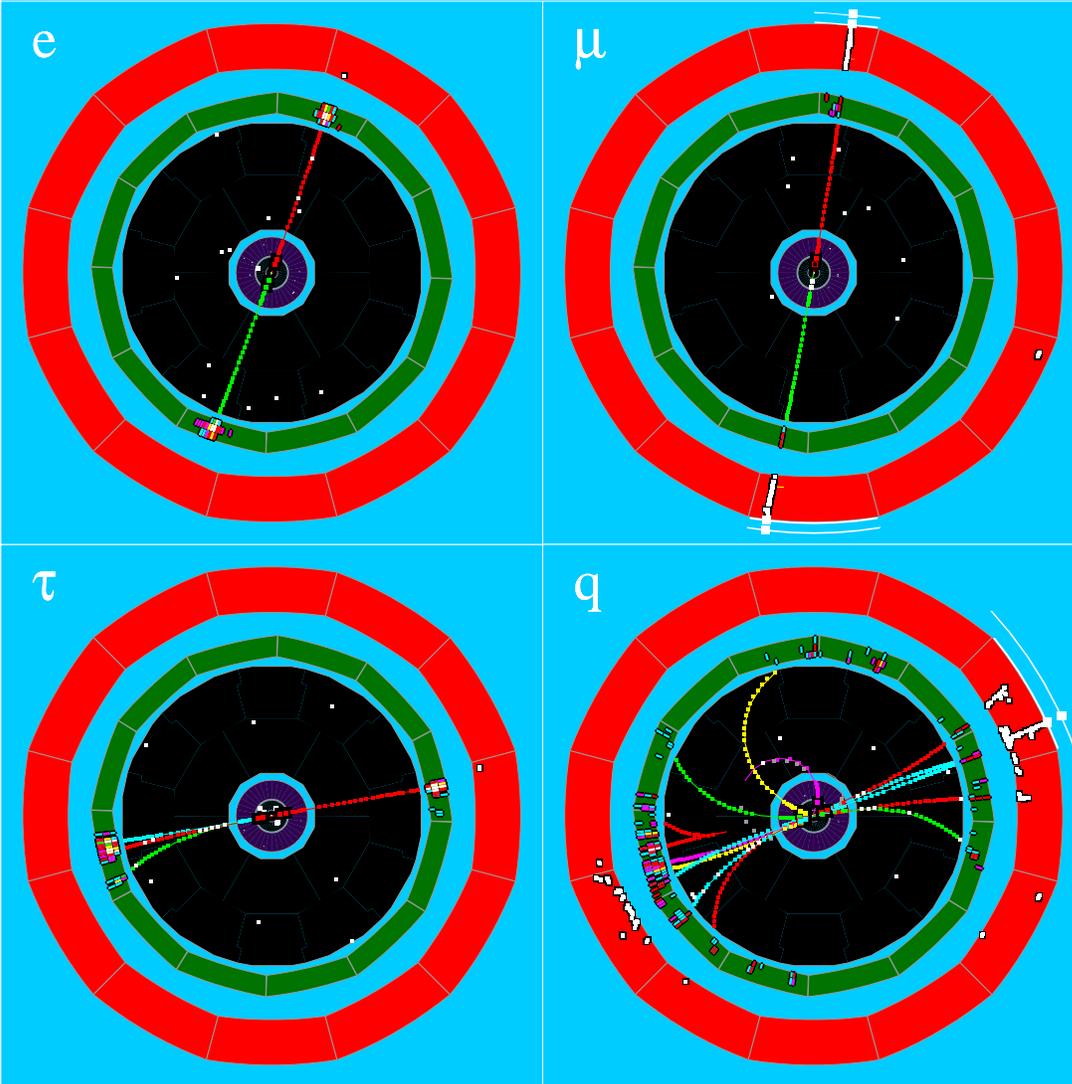
Fig. 2: *The four types of Z boson decays. Up left a decay into a pair of electrons, up right into a pair of muons, bottom left a pair of tau leptons, one decaying into an electron and the other into three particles, bottom right a pair of quarks that fragments into a number of hadrons.*

The sensitivity of N_ν to R_ℓ is small, as there is a cancellation between the two terms containing this quantity. As a result the experimental measurement that enters most in the determination of N_ν is the peak cross section, as already guessed intuitively from Figure 1. This explains how quickly the number of neutrinos was obtained, a few weeks after the start-up of LEP and SLC.

4 Precision measurements of the mass and width of the Z

The interest of precise measurements of the Z line shape parameters is evident when considering the Standard Model expressions for the Z partial widths:

$$\Gamma_f = \frac{G_F M_Z^3}{6\pi\sqrt{2}} \left(a_f^2 + v_f^2\right) N_c \left(1 + \frac{3Q_f^2}{4}\frac{\alpha}{\pi}\right) [1 + \alpha_s/\pi + ...] , \qquad (6)$$

where the couplings a_f, v_f are defined in the contribution by F. Boudjema *et al.*. In Eq. (6) N_c is the number of colours (1 for leptons and 3 for quarks) and the last term between brackets is the QCD perturbative expansion which only applies to quarks. Electroweak corrections to these formulae are
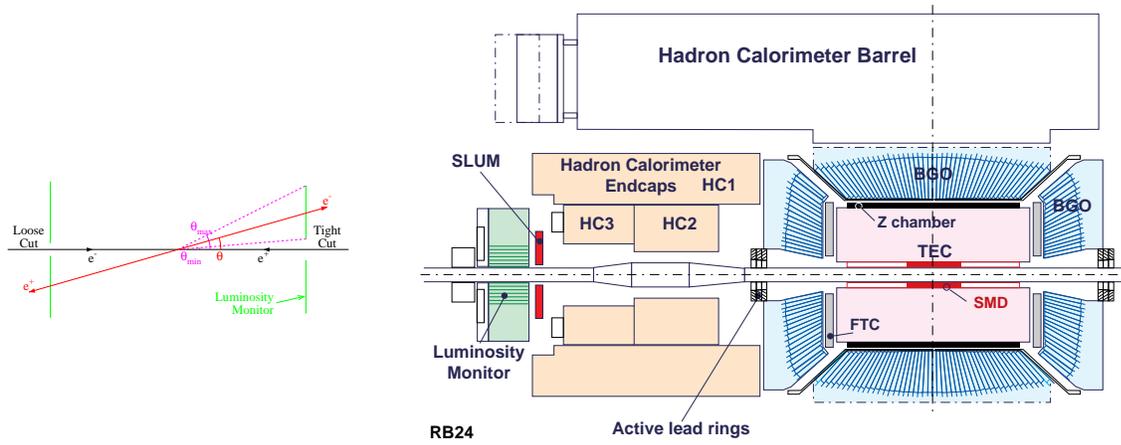
Fig. 3: *Example of a LEP luminosity monitor (the L3 experiment). The process to be detected is elastic $e^+e^- \rightarrow e^+e^-$ scattering, shown on the left, which is seen by a coincidence of an electron in each of the detectors placed in the forward regions of the detector as shown on the right. In order to determine the solid angle seen by this coincidence in a way which is independent of the exact location of the collision point (which is difficult to determine) a set of two different acceptances is defined for the two arms. The precise definition of the acceptance is obtained by a precisely machined silicon telescope (SLUM) positioned in front of the electron calorimeters (luminosity monitor); there is a symmetric device on the other side of the detector.*

largely accounted for by using universal effective couplings at the Z energy scale, both for $\alpha \rightarrow \alpha(M_Z^2)$ for the QED coupling constant, $\alpha_s \rightarrow \alpha_s(M_Z^2)$ for the strong coupling constant and for the weak mixing angle $\sin^2\theta_w \rightarrow \sin^2\theta_w^{\text{eff}}$. These corrections amount to 6% for $\alpha(M_Z^2)$, and to 1-2% for $\sin^2\theta_w^{\text{eff}}$ due to the large mass of the top quark. Small additional non-universal corrections (vertex corrections) amount to a few 10^{-3}; they are insensitive to such effects as the top quark or Higgs boson masses. The b partial width constitutes with a well-known exception, since the vertex correction involving the top quark amounts to 2%.

The interest of the various parameters extracted from the line shape is then as follows:
– The Z mass is one of the precise inputs to the electroweak theory calculations
– The Z width is sensitive to the strong coupling constant and to the electroweak corrections involving the top quark mass, including the b quark vertex correction
– The peak cross section for hadrons is very sensitive to the partial width of the Z into invisible modes, in particular N_ν
– The ratio of hadrons to leptons is very sensitive to the strong couling constant $\alpha_s(M_Z^2)$, and to a lesser extent to the b vertex correction; for different lepton species, they constitute an essential test of the universality of the couplings of the Z boson.

By combination of these measurements one can obtain one of the most precise measurements of $\alpha_s(M_Z^2)$, and more importantly a prediction for the mass of the top quark from radiative corrections. For instance the Z width varies by 2 MeV/c^2 if the top quark mass varies by 10 GeV/c^2.

In 1992, searches for the top quark mass in high energy hadron colliders had not been succesful yet. For this reason, there was great interest in performing precise measurement of the Z width. This can be done by measuring e^+e^- cross sections across the Z resonance. The choice of points and the amount of running necessary at each point was subject to careful studies:
– The best scan would correspond to taking data at the peak, and at two points situated at +2 GeV and -2 GeV from the peak, with nearly equal amounts of data taken at each point.
– The beam energies were chosen so as to allow precise measurement of the LEP beam energies by the technique of resonant depolarisation. To allow build up of the transverse polarisation by the Sokolov-Ternov effect, the energy had to be such that the spin tune was near a half-integer (see the contribution by J. P. Koutchouk *et al.*). By a lucky coincidence, the Z peak happens to correspond to $\nu = 103.5$.
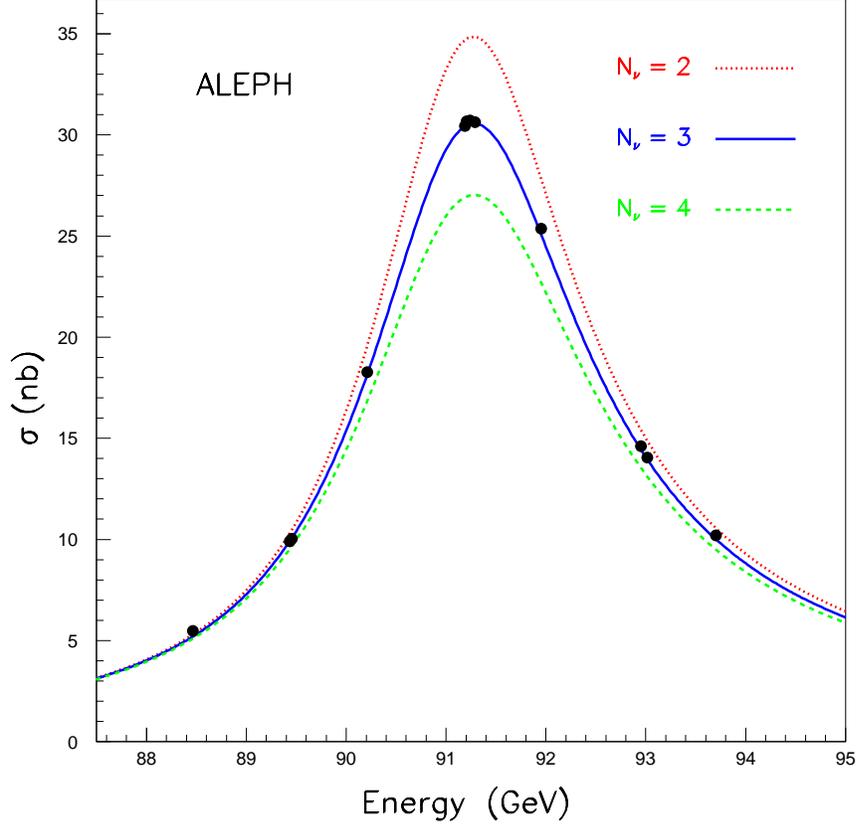
Fig. 4: *The $e^+e^- \to hadrons$ cross section as a function of center-of-mass energy, as measured by ALEPH. The curves represent the Standard Model predictions for two, three and four species of light neutrinos. It is clear from this picture that there is no further light neutrino species with couplings identical to the first three.*

It was thus decided to scan the Z peak at energies corresponding to spin tunes of 101.5, 103.5, 105.5. One unit of spin tune corresponding to 0.44065 GeV beam energy, or 0.8813 in center-of-mass energy, this is close enough to ± 2 GeV.

A very systematic scan of the Z resonance was performed in 1993, with regular energy calibrations of the LEP energies. By that time the dominant source of variations in the LEP energy had been identified to be the effect of terrestrial tides, so that the energy calibration was good to better than 2 MeV.

In addition to this careful determination of the LEP energies, it was necessary to ensure high precision for the experimental determination of the cross sections, with particular attention to possible energy-dependent errors. High statistics were by then available, each experiment collecting around one million events. Major sources of systematic errors on the determination of the Z mass and width in the experiments are: the contamination of the Z decays by non-resonant backgrounds, which tends to widen the apparent shape of the peak; and, the fact that the effect of initial-state radiation is not the same above the peak, where it is dominated by emission of a photon back to the Z peak, and below, where its effect is essentially to reduce the visible cross section.

Nevertheless the experiments were able to produce for the 1994 winter conferences a combined measurement of the Z width with a precision of 3 MeV/c^2, which was the main ingredient to a prediction of the top quark mass of (172^{+13+18}_{-14-20}) GeV/c^2 [11] (the first error corresponds to the experimental error, the second one to the unknown mass of the Higgs boson). A month later, the first observations of the top quark with a mass of (176 ± 16) GeV/c^2 were announced by the CDF [12] collaboration at the Tevatron proton-antiproton collider!

If the Z width measurement is sensitive to relative point-to-point errors, the Z mass is sensitive to the absolute calibration of the beam energy. It was realized in 1994 that the time evolution of the

LEP energy was not as stable as one had expected: jumps were observed during a long-term stability experiment. This may have biased the Z mass measurement, since, during the 1993 scan, the energy calibrations were always performed at the end of data taking periods of 8-10 hours. To monitor this effect, 16 NMR probes were inserted in a sample of the LEP magnets, to observe possible time variations. This should allow a complete study of these jumps, and a new scan was decided for 1995, in conditions similar to those of 1993, and with additional monitoring. In 1994 data had been taken at the Z peak to accumulate large statistics.

As it turned out, the cause of the constant rise in the magnetic field was attributed to electric perturbations due to the passage of the French trains (TGV) in the area. This led to a shift of about 3 MeV in the beam energy, and the corresponding correction to the Z mass, admittedly a larger number than the quoted systematic error from the 1993 scan.

It took several years of analysis by the LEP experimenters to extract the final values of the Z line shape parameters, as given in Table 3. In the end, the Z mass was measured with a precision of $2 \ 10^{-5}$, the Z width, peak cross section and ratio of hadrons to leptons to 10^{-3}, and the relative couplings of the various leptons to a precision of $2 \ 10^{-3}$. This precision exceeds the pre-LEP expectations by one order of magnitude. Such a precision, obtained after many years of hard work and clever tricks, was possible at LEP due to the extremely clean conditions, and the availability of resonant depolarization for the energy calibration in a circular machine. It is unlikely that, at least for the Z mass and witdh, these measurements will be improved in any forseeable future.

Quantity	Main experimental issues	Physics output	Latest measurement
M_Z [GeV/c^2]	**Absolute energy scale** relative cross sections line shape fit (QED rad. corr.)	input to electroweak theory	91.1876 ± 0.0021
Γ_Z [GeV/c^2]	**Relative energy scale** relative cross sections line shape fit (QED rad. corr.)	top and Higgs mass strong coupling constant b vertex correction	2.4952 ± 0.0023
$\sigma_{\text{had}}^{\text{peak},0}$ [nb]	**Absolute cross sections**	$N_\nu \cdot \frac{\Gamma_{\text{inv}}}{\Gamma_{\ell\ell}}$	41.541 ± 0.037
$R_\ell \equiv \frac{\Gamma_{\text{had}}}{\Gamma_{\ell\ell}}$	lepton, hadron event selection	α_s, b vertex correction	20.767 ± 0.025
R_ℓ for electrons	event selection	universality	20.804 ± 0.050
R_ℓ for muons	event selection	universality	20.785 ± 0.033
R_ℓ for taus	event selection	universality	20.764 ± 0.045

Table 3: *Synopsis of parameters of the Z line shape. R_ℓ is defined as $R_\ell \equiv \Gamma_{\text{had}}/\Gamma_{\ell\ell}$, where $\Gamma_{\ell\ell}$ refers to the partial width into a pair of massless charged leptons. The values of this quantity obtained from the separate measurements with electrons, muons and taus are also given.*

References

[1] S. Weinberg, it Gravitation and cosmology, John Wiley ed., 1972.

[2] D. Denegri, B. Sadoulet, M. Spiro, Review of Modern Physics A (1989).

[3] J.R. Ellis, "Zedology", in the LEP summer study, CERN Yellow report 79-01 (1979) 618.

[4] G. S. Abrams et al, (MarkII Coll.) Phys. Rev. Lett. 63 (1989) p. 724.

[5] D. Decamp et al, (ALEPH Coll.) Phys. Lett. B231 (1989) p. 519.

[6] P. Aarnio et al, (DELPHI Coll.) Phys. Lett. B231 (1989) p. 539.

[7] B. Adeva et al, (L3 Coll.) Phys. Lett. B231 (1989) p. 509.

[8] M. Z. Akrawy et al, (OPAL Coll.) Phys. Lett. B231 (1989) p. 530.

[9] G. S. Abrams et al, (MarkII Coll.) Phys. Rev. Lett. 63 (1989) p. 2173.

[10] The LEP collaborations prepare regularly an update of the most precise results. See `http://lepewwg.cern.ch/LEPEWWG` and CERN-EP/2001-098, hep-ex/011-2021 (Dec. 2001). This contains references to the latest experimental and theoretical papers.

[11] B. Pietrzyk, XXIX Rencontres de Moriond, 'Electroweak Interactions and UNified Theories', J. Tran Than Van ed., ed. Frontières, p. 137.

[12] F.Abe et al, (CDF Coll.) Phys. Rev. Lett. 74 (1995) 2626;
D0 Collab., Phys. Rev. Lett. 74 (1995) 2632.